2020

MATHEMATICS (Honours)

Paper Code : II - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :

  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : II - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. Interior of the set $\mathbb{Z}$ is

   A. $\varnothing$

   B. $\mathbb{Z}$

   C. $\mathbb{R}$

   D. None of these.

2. The set $\{\frac{1}{n} : n \in \mathbb{N}\}$ is

   A. open in $\mathbb{R}$

   B. dense in $\mathbb{R}$

   C. closed in $\mathbb{R}$

   D. None of these.

3. $\lim_{n\to\infty}(1 + \frac{1}{3n})^n$ is

   A. e

   B. e^3

   C. $e^{\frac{1}{3}}$

   D. None of these.

4. Let $u_n = \frac{1}{n}$ and $v_n = \frac{(-1)^n}{n}$, where $n \in \mathbb{N}$. Then which of the following is true?

A. $\sum_{n=1}^{\infty} u_n$ is convergent and $\sum_{n=1}^{\infty} v_n$ is divergent

B. $\sum_{n=1}^{\infty} u_n$ is divergent and $\sum_{n=1}^{\infty} v_n$ is convergent

C. both the series $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are convergent

D. both the series $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are divergent.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Which of the following is always true?

A. f is bounded

B. f maps interval to interval

C. f is uniformly continuous

D. f maps open set to open set.

6. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ is such that $|f(x) - f(y)| \leq (x - y)^4$, for all $x, y \in \mathbb{R}$. Then for any $x \in \mathbb{R}$, the value of $f'(x)$ is

A. 0

B. 1

C. 2

D. 4.

7. The expression $f(x) = e + e(x-1) + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \cdots$ is the Taylor series for

A. $f(x) = x$ about $x = e$

B. $f(x) = x^2$ about $x = -1$

C. $f(x) = x^2$ about $x = 1$

D. $f(x) = e^x$ about $x = 1$.

8. The value of $\int_0^{\infty} e^{-x^2} dx$ is

A. 0

B. $\sqrt{\pi}$

C. $\frac{\sqrt{\pi}}{2}$

D. None of these.

9. The value of $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

A. is 0

B. is 1

C. is -1

D. does not exist.

10. Radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $(\frac{\pi}{2}, 4)$ is

A. $\frac{5\sqrt{5}}{4}$

B. $\frac{5\sqrt{5}}{2}$

C. $5\sqrt{5}$

D. 1.

2020

MATHEMATICS (Honours)

Paper Code : II - B

[New Syllabus]

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A

(35 Marks)

Answer any **seven** questions

$5 \times 7 = 35$

1. Prove that for any positive real number x, there exists a natural number n such that $0 < \frac{1}{n} < x$.
   Find the derived set of the set $A = \{m + \frac{1}{n} : m, n \in \mathbb{N}\}$. [3+2]

2. Prove that the sequence $\{x_n\}$ defined by
   $x_1 = \sqrt{6}$,
   $x_{n+1} = \sqrt{6 + x_n}$, for $n \geq 1$
   is convergent. Hence find the limit. [3+2]

3. Show that the sequence $\{x_n\}$ defined by $x_n = (1 + \frac{1}{n})^n$ is convergent and $\lim\limits_{n \to \infty} x_n$ lies between 2 and 3. [5]

4. Prove that an absolutely convergent series is convergent. Hence test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$. [3+2]

5. Give definition of conditionally convergent series. Test the convergence of the series $\dfrac{(\log 2)^2}{2^2} + \dfrac{(\log 3)^2}{3^2} + \dfrac{(\log 4)^2}{4^2} + \cdots + \dfrac{(\log n)^2}{n^2} + \cdots$. [1+4]

6. Let f, g be defined on $A \subseteq \mathbb{R}$ to $\mathbb{R}$, and let c be a cluster point of A. Suppose that f is bounded on a neighbourhood of c and that $\lim_{x \to c} g(x) = 0$. Prove that $\lim_{x \to c} f(x)g(x) = 0$.

   Hence show that $\lim_{x \to 0} x^2 \cos \frac{2}{x} = 0$. [3+2]

7. A real valued function f is continuous on $[0, 2]$ and $f(0) = f(2)$. Prove that there exists at least one point c in $[0, 1]$ such that $f(c) = f(c + 1)$. [5]

8. Prove that the equation $(x-1)^3 + (x-2)^3 + (x-3)^3 + (x-4)^3 = 0$ has only one real root. [5]

9. (a) Evaluate $\lim_{x \to 0} (\cos x)^{\cot^2 x}$.

   (b) A real valued function f is defined on some neighbourhood of a and f is differentiable at a. Prove that $\lim_{h \to 0} \dfrac{f(a+h) - f(a-h)}{2h} = f'(a)$. [3+2]

10. Prove that $0 < \dfrac{1}{\log(1+x)} - \dfrac{1}{x} < 1$, for $x > 0$. [5]

Group-B
(20 Marks)

Answer any **four** questions

. $5 \times 4 = 20$

11. Use ϵ-δ definition to show that the function

$$f(x, y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

   is continuous at the origin. [5]

12. Let

$$f(x, y) = \begin{cases} y \sin \frac{1}{x} + \frac{xy}{x^2+y^2}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

   Verify that $\lim_{x \to 0} \lim_{y \to 0} f(x, y)$ exists but neither $\lim_{(x,y) \to (0,0)} f(x, y)$ nor $\lim_{y \to 0} \lim_{x \to 0} f(x, y)$ exists. [1+3+1]

13. Determine whether the function $f(x, y) = \sqrt{|xy|}$ is differentiable at the origin. [5]

14. If $\frac{x^2}{a+u} + \frac{y^2}{b+u} + \frac{z^2}{c+u} = 1$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right).$$

[5]

15. If

$$u^3 + v + w = x + y^2 + z^2,$$
$$u + v^3 + w = x^2 + y + z^2$$
$$\text{and } u + v + w^3 = x^2 + y^2 + z,$$

then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1 - 4(xy + yz + zx) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2v^2w^2}.$$

[5]

16. Let $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}.$$

[5]

Group-C
(25 Marks)
Answer any **five** questions

. $5 \times 5 = 25$

17. Find the volume of the solid formed by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis. [5]

18. Show that the radius of curvature of the envelope of the line $x\cos\theta + y\sin\theta = f(\theta)$ is $f(\theta) + f''(\theta)$. [5]

19. Find the asymptotes of the polar curve $r = \frac{a\theta}{\theta - 1}$. [5]

20. Find the area included between the curves $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$. [5]

21. Evaluate:

$$\int_0^1 \frac{dx}{(1 - x^6)^{\frac{1}{6}}}$$

[5]

22. Find the centre of gravity of the area in the first quadrant bounded by the parabola $y^2 = 4ax$, the latus rectum and the x-axis. [5]

23. Find the pedal equation of the cardioid $r = a(1 + \cos\theta)$. [5]