2020

MATHEMATICS (General)

Paper Code - MTMGII-DC-2/GE-2

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) State the least upper bound axiom for the set of real numbers.

(b) Give an example of a bounded sequence in $\mathbb{R}$ which is not convergent.

(c) Examine the applicability of Rolle's theorem in $[-1, 1]$ for the function $f(x) = |x|$.

(d) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{x}(1+x)}$.

(e) Find the differential equation of the family of circles $(x-\alpha)^2 + y^2 = 4$, where α is a parameter.

(f) Find the order and degree of the differential equation $\sqrt{1+(\frac{dy}{dx})^2} = 1 + x$.

(g) Find the value of $\int_0^\infty e^{-x^2} dx$, assuming $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Group - B

(10 Marks)

Answer any *two* questions. 5×2=10

2. Let $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$. Show that $\{x_n\}$ is a real convergent sequence. 5

3. Using Lagrange's MVT, show that $\frac{x}{1+x} < \log(1+x) < x$ for $x > 0$. 5

4. (a) Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$. 3

   (b) Hence or otherwise evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x dx$. 2

5. Solve : $(D^2 + 3D + 2)y = e^{2x} \sin x$, where $D \equiv \frac{d}{dx}$. 5

Group - C

(18 Marks)

Answer any *two* questions. 9×2=18

6. (a) Expand the function $f(x) = \log(1+x)$, $-1 < x < \infty$ about $x = 2$ by Taylor's formula with Lagrange's form of remainder after three terms. 4

   (b) Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{7}$ and $u_{n+1} = \sqrt{7 + u_n}$ converges to the positive root of the equation $x^2 - x - 7 = 0$ 5

7. (a) Show that $\int_0^{\infty} e^{-x}$ is convergent. Hence show that $\int_0^{\infty} \frac{dx}{e^x + 1}$ is convergent. 3+2

   (b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$$x^2 y_{n+2} + (2n-1)x y_{n+1} + (n^2+1)y_n = 0.$$

4

8. (a) Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. 4

(b) Solve: $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10(x + \frac{1}{x})$. 5
