2021

MATHEMATICS (Honours)

Paper Code : VII - A & B

(New Syllabus)

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : VII - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Each question carries 2 marks.

Notations and symbols have their usual meanings.

1. Let $\overrightarrow{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, S be the surface of the sphere $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be the inward unit normal vector to S. Then $\iint\limits_S \overrightarrow{F} \cdot \hat{n} dS$ is equal to

   A. 4π

   B. -4π

   C. 8π

   D. -8π

2. Let C be the circle $x^2 + y^2 = 1$ taken in the anti-clockwise sense. Then the value of the integral $\int\limits_C [(2xy^3 + y)dx + (3x^2y^2 + 2x)dy]$ is

   A. 1

   B. $\pi/2$

   C. π

   D. 0

3. The work done by the force $\overrightarrow{F} = 4y\hat{i} - 3xy\hat{j} + z^2\hat{k}$ in moving a particle over the circular path $x^2 + y^2 = 1$, $z = 0$ from $(1, 0, 0)$ to $(0, 1, 0)$ is

   A. $\pi + 1$

   B. $-\pi - 1$

   C. $-\pi + 1$

   D. $\pi - 1$

4. The number of degrees of freedom of a rigid body is

A. 9

B. 3

C. 6

D. 1

5. A uniform solid cylinder rolls down along an inclined plane, inclined at an angle α with the horizon, rough enough to prevent any sliding. For pure rolling, we must have

A. $\mu > \frac{2}{7}\tan\alpha$

B. $\mu > \frac{1}{3}\tan\alpha$

C. $\mu > \frac{1}{2}\tan\alpha$

D. $\mu > \frac{2}{5}\tan\alpha$

6. The minimum force P required to drag a heavy body of weight W along a rough horizontal plane is (Given: μ is the coefficient of friction, λ is the angle of friction)

A. $P = W\sin\lambda$

B. $P = W\cos\lambda$

C. $P = W\tan\lambda$

D. $P = W\sec\lambda$

7. A uniform cubical box of edge a is placed on the top of a fixed sphere. Then the least radius of the sphere for which the equilibrium is stable is

A. $\frac{a}{3}$

B. $\frac{a}{2}$

C. $\frac{a}{4}$

D. $\frac{a}{5}$

8. The co-ordinates $(\overline{x}, \overline{y})$ of c.g. of a circular arc making an angle 2α at the centre are

A. $(\frac{a \sin \alpha}{\alpha}, 0)$

B. $(\frac{2}{3}\frac{a \sin \alpha}{\alpha}, 0)$

C. $(0, \frac{2}{3}\frac{a \cos \alpha}{\alpha})$

D. $(0, \frac{2}{3}\frac{a \tan \alpha}{\alpha})$

9. The moment of inertia of a hollow sphere (i.e. thin spherical shell) of mass M and radius a about any diameter is

A. $\frac{2}{5}Ma^2$

B. $\frac{2}{3}Ma^2$

C. $\frac{1}{5}Ma^2$

D. $\frac{1}{3}Ma^2$

10. If K is the radius of gyration of a rigid body of mass M about an axis, then the kinetic energy of the rigid body rotating with constant angular velocity about the axis is

A. $\frac{1}{2}MK^2\omega$

B. $MK^2\omega$

C. $MK^2\omega^2$

D. $\frac{1}{2}MK^2\omega^2$

2021

MATHEMATICS (Honours)

Paper Code : VII - B

(New Syllabus)

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A
(10 Marks)

Answer any *two* questions.

1. Verify Green's theorem for

$$\int_C \left[(3x-8y^2)dx+(4y-6xy)dy\right],$$

where C is the boundary of the region bounded by $x=0, y=0$ and $x+y=1$. 5

2. Use divergence theorem to evaluate

$$\int_S \overrightarrow{A}\cdot d\overrightarrow{S},$$

where S is the surface of the sphere $x^2+y^2+z^2=a^2$ and $\overrightarrow{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$. 5

3. Evaluate the surface integral

$$\int_S (\overrightarrow{F}\cdot\hat{n})dS,$$

where $\overrightarrow{F}=x\hat{i}-y\hat{j}+(z^2-1)\hat{k}$ and S is the cylinder formed by $z=0$, $z=1$, $x^2+y^2=4$. 5

4. Verify Stokes' theorem for $\overrightarrow{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$, where S is the surface of the cube $x = 0,\ y = 0,\ z = 0,\ x = 2,\ y = 2,\ z = 2$ above the xy-plane. 5

Group-B
(25 Marks)

Answer question no. 5 and any *three* from the rest.

5. Answer any *one* question $4 \times 1 = 4$

   (a) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$. Find the equation of the line of action of their resultant.

   (b) A heavy elastic string whose natural length is $2\pi a$ is placed round a smooth cone whose axis is vertical and whose semi-vertical angle is α. If W is the weight and λ is the modulus of elasticity of string, prove that it will be in equilibrium when in form of a circle whose radius is $a\left(1 + \frac{W}{2\pi\lambda}\cot\alpha\right)$.

6. A solid homogeneous hemisphere rests on a rough horizontal plane and against a rough vertical wall, the coefficients of friction being μ and μ' respectively. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu}{3}\frac{1+\mu'}{1+\mu\mu'}\right)$. 7

7. A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1} 3/8$, and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable. 7

8. Two equal forces act along the generators of the same system of the hyperboloid $\frac{x^2+y^2}{a^2} - \frac{z^2}{b^2} = 1$ and cut the plane $z = 0$ at the extremities of perpendicular diameters of the circle $x^2 + y^2 = a^2$; show that the pitch of the equivalent wrench is $\frac{a^2 b}{a^2+2b^2}$. 7

9. Find the position of the c.g. of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ bounded by the principal planes when the density at a point (x, y, z) is kxy, where k is a constant. 7

10. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, then prove that $\tan\phi = \frac{3}{8} + \tan\theta$. 7

Group-C
(25 Marks)

Answer question no. 11 and any *three* from the rest.

11. Answer any *one* question $4 \times 1 = 4$

(a) Prove that the sum of moment of inertia of a rigid body about any three perpendicular lines is constant.

(b) Prove that the moment of momentum of a body moving in two dimensions about the origin is $Mvp + Mk^2\frac{d\theta}{dt}$.

12. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α, show that $\omega^2 = \frac{3g}{4a\cos\alpha}$. Prove also that the direction of reaction at the hinge makes with the vertical an angle $\tan^{-1}(\frac{3}{4}\tan\alpha)$. 4+3

13. An elliptic lamina can rotate about a horizontal axis passing through a focus and perpendicular to its plane. If the eccentricity of the ellipse is $\sqrt{\frac{2}{5}}$, then show that the centre of oscillation will be at the other focus. 7

14. A circular homogeneous plate is projected up a rough inclined plane with velocity V with no rotation, the plane of the plate being in the plane of greatest slope. Show that the plate stops sliding after a time $\frac{V}{g(3\mu\cos\alpha+\sin\alpha)}$, where μ is the coefficient of friction and α is the inclination of the plane with the horizon. 7

15. Two equal uniform rods AB and AC are freely jointed at A. They are placed on a table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to AC. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2:7. 7

16. A rough cylinder, of mass M, is capable of motion about its axis which is horizontal; a particle of mass m is placed on it vertically above the axis and the system is slightly disturbed. Show that the particle will slip on the cylinder when it has moved through an angle θ given by $\mu(M + 6m)\cos\theta - M\sin\theta = 4m\mu$. 7

Group-D
(20 Marks)

Answer question no. 17 and any *two* from the rest.

17. Answer any *one* question $6 \times 1 = 6$

    (a) Prove that the pressure at a point in a fluid in equilibrium is the same in all directions.

    (b) A circular tube is half full of liquid and is made to revolve round a vertical tangent line with angular velocity ω. If a is the radius of the tube, then prove that the diameter passing through the free surfaces of the liquid is inclined at an angle $\tan^{-1}(\frac{\omega^2 a}{g})$ to the horizon.

18. A closed right circular cylinder is very nearly filled with water and is made to rotate about its axis which is vertical. If the angular velocity is $\frac{\sqrt{2gh}}{a}$, then show that the whole thrust on the base is half as much again when the liquid is at rest, where h is the height and a is the radius of the cylinder. 7

19. A vertical circular cylinder of height $2h$ and radius r, closed at the top, is just filled by equal volumes of two liquids of densities ρ and σ, $(\sigma > \rho)$. If the axis gradually inclined to the vertical, then show that the pressure at the lowest point of the base will never exceed $g(\rho + \sigma)(r^2 + h^2)^{1/2}$. 7

20. A semi-circular lamina is completely immersed in water with its plane vertical, so that the extremity A of its bounding diameter is in the surface, and the diameter makes with its surface an angle α. If E is the centre of pressure and ϕ is the angle between AE and the diameter, then prove that $\tan\phi = \frac{3\pi + 16\tan\alpha}{16 + 15\pi\tan\alpha}$. 7

21. A cylindrical piece of wood of length l and sectional area α is floating with its axis vertical in a cylindrical vessel of sectional area A which contains water. Prove that the work done in slowly pressing down the wood until it is completely immersed is $\frac{1}{2}gl^2\alpha(1-\frac{\alpha}{A})\frac{(\rho-\sigma)^2}{\rho}$, where ρ and σ are the densities of water and wood respectively. 7
