

# GOUR MAHAVIDYALAYA

## MATHEMATICS (General)

Paper Code: MATH-DC-02/GE-02(Internal)

[New Syllabus]

Full Marks: 20

Time: One Hour

*Notations and symbols have their usual meanings*

1. Answer any four questions. 4 × 5 = 20
- (a) State Cauchy's general principal of Convergence of a real sequence. Use Cauchy's general principal of convergence to prove that the sequence  $\{x_n\}$  where  $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ , is not convergence. [2+3=5]
- (b) Test the convergence of the series  $1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \dots$  [5]
- (c) State Rolle's theorem. Verify Rolle's theorem of the function  $f(x) = x^2 + \cos x$  on  $[-\frac{\pi}{4}, \frac{\pi}{4}]$ . [2+3=5]
- (d) If  $I_n = \int \frac{\sin nx}{\sin x} dx$ , show that  $(n - 1)\{I_n - I_{n-2}\} = 2 \sin(n - 1)x$  [5]
- (e) Examine the convergence of the improper integral  $\int_0^{\infty} \frac{dx}{x^{\frac{1}{2}}(1+x^{\frac{1}{4}})}$  [5]
- (f) Find integrating factor of the differential equation  $(xy^2 - e^{1/x^3})dx - x^2ydy = 0$ , then solve it. [2+3=5]
- (g) Solve the differential equation  $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$  [5]
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