U.G. 5th Semester Examination 2021

MATHEMATICS (Honours)

Paper : DSE-2

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

DSE-1A

[Differential Geometry]

Group - A

(4 Marks)

1. Answer any ***four*** questions : 4×1=4

(a) If $i, j = 1, 2, ...n$. $\sum \delta_{ij} = ?$

(b) Given A^i are functions of (x^1, x^2,....x^n). Where $A^i B_i$ is an invariant. If B_i be a covariant vector what is A^i, justify.

(c) Describe Riemannian space.

(d) Difine binormal of a space curve.

(e) Parametrize the unit circle $x^2 + y^2 = 1$.

(f) Write the cannonical geodesic equation.

(g) What is the curvature of a unit circle.

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. Calculate tangent vector (T), principal normal vector (N), Binormal (B) of the curve : $\alpha(t) = (\cosh t, \sinh t, t)$.

3. Calculate any two chirtoffel symbols for the space curve $x(u,v) = (u\cos v, u\sin v, u)$.

4. Show that covarient derivative of g_{ij} and δ_{ij} is zero.

5. Show that in S_n, a symmetric covariant tenson of order two has at most $\frac{n(n+1)}{2}$ different components.

Group - C

(18 Marks)

Answer any ***two*** questions : 2×9=18

6. (a) What are the symmetric and skew-symmetric tensor. Show that any tensor of typ (0, 2) is the sum of a symmetric and an skew-symmetric tensor. 2+3

   (b) Show that $A_{i,j}$ the covariant derivative of a covariant tensor is a tensor of type (0, 2). 4

7. State and prove serret-Frenet formula. 9

8. (a) Deduce the equation of Geodesic. 5

   (b) If the metric is given by $ds^2 = 5\left(dx^1\right)^2 + 3\left(dx^2\right)^2 + 4\left(dx^3\right)^2 - 6dx^1dx^2 + 4dx^2dx^3$

   Evaluate : (i) g and (ii) g^{ij} 4

DSE-2B
[Fluid Mechanics]

Full Marks : 32 Time : 2 Hours

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Group - A
(4 Marks)

1. Answer any ***four*** questions : 4×1=4

   (a) Explain the terms Perfect fluid and pressure at a point in a fluid.

   (b) Prove that, the pressure at any point within liquid is given by $p = hfpg$.

   (c) What are the difference between Lagrangian and Eulerian method?

   (d) State Pascal's law.

   (e) Discuss steady and unsteady flow with example.

   (f) Test whether the motion specified by $\vec{q} = \dfrac{\lambda^2\left(x\hat{j} - y\hat{i}\right)}{x^2 + y^2}$ (λ = constant)

   is a possible motion for an incompressible fluid.

   (g) State Reynolds transport theorem.

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. A fine tube bent in the form of an ellipse is held with its plane vertical and its filled with n liquids whose densities are $\rho_1, \rho_2 \ldots\ldots\ldots, \rho_n$ taken in order round the elliptic tube. If $r_1, r_2, \ldots\ldots\ldots, r_n$ be the densities of the points of separation from either focus, Prove that $r_1(\rho_1 - \rho_2) + r_2(\rho_2 - \rho_3) + \ldots\ldots + r_n(\rho_n - \rho_1) = 0$. 5

3. A Semi-circular area is completely immersed in water with its plane vertical, so that the extremity A of its bounding diameter is in the surface and the diameter makes with the surface an angle α. Prove that if E be the C.P. and θ the angle between AE and the diameter, $\tan\theta = \frac{3\pi + 16\tan\alpha}{16 + 15\pi\tan\alpha}$ 5

4. Obtain the fundamental equation in the form grad $p = \int \vec{F}$ for a fluid in equilibrium under a given system of external forces $\vec{F}$ per unit mass of the fluid. Hence show that the necessary condition of equilibrium is $\vec{F}$. Curl $\vec{F} = 0$.

5. Define the equation of continuity. Obtain an expression for continuity equation for a three dimensional steady incompressible flow. 5

Group - C
(18 Marks)

Answer any ***two*** questions : 2×9=18

6. (a) Show that the pressure at a point in a fluid in equilibrium is the same in every direction. 4

   (b) Show that the pressure at a small depth z below the surface of a sphere of water attracted to the center of the sphere with a force producing an acceleration $\frac{\mu}{r^2}$ at a distance r approximately $\pi + \rho g\left(z + \frac{z^2}{a}\right)$, where a is the radius of the sphere and g the attraction of unit mass at the surface of the sphere. 5

7. (a) Show that the depth of the centre of pressure of a plane area immersed in a liquid is greater than the depth of its centre of gravity. 4

   (b) Show that the forces represented by $X = \mu\left(y^2 + yz + z^2\right), Y = \mu\left(z^2 + zx + x^2\right), Z = \mu\left(x^2 + xy + y^2\right)$ will keep a mass of liquid at rest, if the density $\propto \frac{1}{(\text{distance})^2}$ from the plane $x + y + z = 0$, and the curves of equal pressure and density will be circles. 5

8. (a) For an incompressible fluid $\vec{q} = (-wy, wx, 0)$ (w = constant), discuss the nature of the flow.

(b) Prove that the acceleration of a fluid particle at P is given by

$$\vec{f} = \frac{\partial \vec{q}}{\partial t} + \text{grad}\left(\frac{1}{2}\vec{q}^2\right) - \vec{q} \times \text{Curl } \vec{q}\,. \qquad 4+5$$

DSE-2C
[Portfolio Optimization]

Full Marks : 32 Time : 2 Hours

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Group - A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

   (a) What is investment Risk?

   (b) What is Ex-post return and Ex-anti return?

   (c) What do you mean by minimum variance portfolio?

   (d) If a portfolio contains 50 securities, determine the total information required under Markowitz Model.

   (e) What is β of security?

   (f) What is Security market line (SML)?

   (g) What is Sharpe's risk Index?

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. Derive the portfolio return and portfolio risk of 2 securities.

3. If return of two assets are perfectly correlated then determine the shape of efficient frontier.

4. Write the difference between CML (Capital Market Line) and SML (Security Market Line).

5. Discuss Jensen's performance measure for Portfolios.

Group - C

(18 Marks)

Answer any ***two*** questions : 2×9=18

6. How do you select the best combination of securities in portfolio for risk minimization?

7. What is Diversification? What is systematic and unsystematic risk?

8. Discuss Eugene Fama's Portfolio Decomposition.

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