U.G. 6th Semester Examinations 2022

MATHEMATICS (Honours)

Paper Code : DC-13

[CBCS]

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

[LINEAR PROGRAMMING & GAME THEORY]

Group-A

1. Answer any ***four*** questions : 1×4=4

(a) How many basic solutions are there in the set of equation :

$$2x_1 - 5x_2 + x_3 + 3x_4 = 4$$

$$3x_1 - 10x_2 + 2x_3 + 6x_4 = 12$$

Justify your answer.

(b) Examine whether $S = \{X = (x, y) / |x| \leq 2\}$ is a convex set or not.

(c) Write the dual of the primal problem :

$$\max\ z = -x_1 + x_2$$

$$\text{subject to } \begin{aligned} 5x_1 + x_2 &\leq 3 \\ x_1 - 9x_2 &\leq 1 \\ x_2 &\geq -1 \end{aligned}$$

where $x_1, x_2 \geq 0$.

(d) Solve the 2 × 2 game by algebraic method :

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{array}{|cc|} \hline 4 & -4 \\ -4 & 4 \\ \hline \end{array} \end{array}$$

(e) Show that the LPP

$$\max\ z = 2x_2 + x_3$$

$$\text{subject to } x_1 + x_2 - 2x_3 \le 7$$

$$-3x_1 + x_2 + 2x_3 \le 3$$

$$x_1, x_2, x_3 \ge 0$$

have an unbounded solution.

(f) Write down the general rules for dominance in a game problem.

(g) What is unbalanced assignment problem? How it can be solved?

Group-B

Answer any ***two*** questions : 5×2=10

2. Find the optimal solution of the following Transportation Problem :

	D_1	D_2	D_3	D_4	a_i
O_1	5	4	6	14	15
O_2	2	9	9	6	4
O_3	6	11	7	13	8
b_j	9	7	5	6	

3. Use two phase simplex method to solve

$$\min\ z = x_1 + x_2 + x_3$$

$$\text{subject to } x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \le 3$$

$$2x_2 + x_3 \ge 4$$

$$\text{and } x_1, x_2, x_3 \ge 0$$

4. Solve the following assignment problem :

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

5. Solve the following 4×3 game whose pay-off matrix is given by

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 6 & -2 & 1 \\ 9 & 15 & 2 \\ 3 & -1 & 4 \\ 7 & 13 & 0 \end{bmatrix} \end{array}$$

Group-C

Answer any ***two*** questions : 9×2=18

6. (a) Solve the following game by graphical method whose pay-off matrix is given by

	B_1	B_2	B_3	B_4
A_1	4	–2	3	–1
A_2	–1	2	0	1
A_3	–2	1	–2	0

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(b) Prove that the transportation problem always has a feasible solution. 3

7. (a) Find the optimal solution of the following L.P.P. by solving its dual :

maximize $z = 3x_1 + 4x_2$

subject to $x_1 + x_2 \le 10$

$2x_1 + 3x_2 \le 18$

$x_1 \le 8$

$x_2 \le 6$

$x_1, x_2 \ge 0$

(b) Obtain an initial basic feasible solution to the transportation problem using matrix minima method :

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

5+4

8. (a) Show that the feasible solution (1, 0, 1, 6) of the system

$$x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 - x_4 = 0$$

is not basic. 4

(b) Solve the following L.P.P. by simplex method :

maximize $z = 2x_1 + 3x_2$

subject to $x_1 + x_2 \leq 8$

$x_1 + 2x_2 = 5$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$ 5