U.G. 4th Semester Examinations 2022

MATHEMATICS (Honours)

Paper Code : DC-09

(Mechanics)

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer any ***four*** questions : 1×4=4

(a) Prove that the distance of the line of action of the resultant force of a system of co-planar forces from the origin is $\frac{G}{R}$.

(b) Find the centre of gravity of a circular are making an angle 2α at the centre.

(c) Define coefficient of friction.

(d) A particle moves with a S.H.M., its position of rest being at a distance a from the centre. Find, by the principle of energy, the velocity at the centre.

(e) A particle falls from a height h in time t upon a fixed horizontal plane. It rebounds and reaches the maximum height h' in time t'. Show that $t' = et$.

(f) Prove that the acceleration of a particle moving in a curve with uniform speed is $\rho\dot{\psi}^2$.

(g) Prove that the velocity from infinity under the attraction F to a point whose distance from the centre of force is r, is given by $v^2 = -2\int_{\infty}^{r} F\,dr$.

Group-B

Answer any ***two*** questions : 5×2=10

2. The altitude of a cone is h and the radius of the base is r; a string is fastened to the vertex and to a point on the circumference of the circular base and is then put over a smooth peg. Show that, if the cone rest with its axis horizontal, the length of the string must be $\left(h^2 + 4r^2\right)^{\frac{1}{2}}$.

3. A rod AB is movable about a point A and to the point B is attached a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. Prove by the principle of virtual work that the horizontal force necessary to keep the ring at rest is $\dfrac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)}$.

4. A particle moves under a force $m\mu\left\{3au^4-2\left(a^2-b^2\right)u^5\right\}(a>b)$ and is projected from an apse at a distance $a+b$ with velocity $\dfrac{\sqrt{\mu}}{a+b}$. Show that its orbit is $r=a+b\cos\theta$.

5. Two bodies m_1 and m_2 are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest; m_2 falls off. Show that the distance of m_1 from the upper end of the string at time t is $a+b+c\cos\left(\dfrac{g}{bt}\right)^{\frac{1}{2}}$, where a is the natural length of the string, b and c $(b>c)$ are the distances by which it would extended when supporting m_1 and m_2 respectively.

Group-C

Answer any ***two*** questions : 9×2=18

6. (a) Forces X, Y, Z act along the three given lines given by the equations $y=0$, $z=c$; $z=0$, $x=a$; $x=0$, $y=b$. Prove that the pitch of the equivalent wrench is $\dfrac{aYZ+bZX+cXY}{X^2+Y^2+Z^2}$. If the wrench reduces to a single force, show that the line of action of the force must lie on the hyperboloid $(x-a)(y-b)(z-c)=xyz$. 5

   (b) A regular hexagon is composed of six equal heavy rods freely jointed together and two opposite angles are connected by a string, which is horizontal, one rod being in contact with a horizontal plane, at the middle point of the opposite rod is placed a weight W_1, if W be the weight of each rod, show that the tension of the string is $\left(3W+W_1\right)/\sqrt{3}$. 4

7. (a) If a system of co-planar forces acting on a rigid body be in equilibrium and the body undergo a slight displacement consistent with the geometrical conditions of the system, prove that the algebraic sum of the virtual works is zero; and conversely, if this algebraic sum be zero, the forces are in equilibrium. 7

   (b) A particle describes the curve $p^2=ar$ under a force F to the pole. Find the law of force. 2

8. (a) Prove that for a particle of mass m falling from rest under gravity from a height h above the ground, the sum of the kinetic energy and the potential energy of the particle is constant at every point of its path. 6

(b) A straight smooth tube revolves with constant angular velocity ω in a horizontal plane about one extremity which is fixed. If at zero time a particle inside it be at a distance a from a fixed end and moving with constant velocity V along the tube, then show that its distance at time t is $a \cos h\, \omega t + \frac{V}{\omega} \sin h\, \omega t$. 3