Gour Mahavidyalaya

MATHEMATICS(General)

Paper Code: DC04/GE04

Semister IV

Numerical Methods and Probability Theory

Full Marks : 32 Time : two hours

(GROUP A)
(4 Marks)

1. Answer any four. 4×1=4

 (a) If $f(x) = x^2 - 117 = 0$ then what is the iterative formula for Newton Raphson Method.

 (b) Show that $\Delta\nabla = \Delta - \nabla$

 (c) What is the number of significant figure of the numbers 0.0320700 and 3200

 (d) Define absolute error and relative percentage error.

 (e) State Baye's theorem.

 (f) Define moments generating function.

 (g) What is mean and variance of Binomial distribution $B(n, p)$.

(GROUP B)
(10 Marks)

Answer any two. 2×5=10

2. Solve the system of equation,
$2x_1 + 3x_2 + 1x_3 = 9$
$15x_1 + 2x_2 + 3x_3 = 6$
$3x_1 + 1x_2 + 1x_3 = 8$
correct up to 3 significant figures. [5]

3. Find $y(4.4)$ by Euler's modified method, taking $h = 0.2$, from the differential equation: $\frac{dy}{dx} = \frac{2-y^2}{5x}, y = 1$ when $x = 4$. [5]

4. Determine the value of k such that f(x) defined by

$$f(x) = \begin{cases} kx(1-x) & ;0 < x < 1 \\ 0 & ;elsewhere \end{cases}$$

is probability density function.
Find the corresponding distribution function. [5]

5. A and B are two independent witness in a case. The probability that A will speak the truth is x and the probability that B will speak the truth is y. A and B agree in a certain statement. Show that the probability that this statement is true is $\frac{xy}{1-x-y-xy}$ [5]

(GROUP C)
(18 Marks)

Answer any two. 2×9=18

6. (a) Compute $f(2)$ from the given table: [5]

x	0	1	3	4
f(x)	5	6	50	105

(b) Find $y(0.2)$ from the differential equation $\frac{dy}{dx} = x - y$, $y(0) = 1$, taking $h = 0.1$, by Runge-Kutta method, correct to five decimal places. [4]

7. (a) If $a(\neq 0), c(\neq 0), b, d$ are constants, prove that

$$\rho(aX + b, cY + d) = \frac{ac}{|a||c|}\rho(X,Y)$$

where $\rho(X,Y)$ is correlation coefficient between X and Y. [5]

(b) Deascribe the method of bisection to find a root of an equation. [4]

8. (a) If m and μ_r denote the mean and central rth moment of a Poisson distribution, then prove that

$$\mu_{r+1} = rm\mu_{r-1} + m\frac{d\mu_r}{dm}$$

[5]

(b) Let A and B be two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. Show that $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$ [4]

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2023

GOUR MAHAVIDYALAYA

DEPARTMENT OF MATHEMATICS

Paper : MATH-G-SEC 02

[CBCS]

Full Marks: 32 Time: Two Hours

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A

4 Marks

1. Answer any **four** questions : [4×1=4]

(a) Is it true that every relation which is symmetric and transitive is also reflexive? Give reasons.

(b) If a set S has m elements and a set T has n elements, find the number of elements in $S \cup T$. Assume that $S \cap T$ has k elements.

(c) Is the set $A = \{x : x + 4 = 4\}$ null? Justify it.

(d) Draw the circuit which realises the Boolean expression $(x + y + z^{'}) \cdot (x + y^{'} + z) \cdot (x^{'} + y + z)$.

(e) Give an example of existential quantifier.

(f) Define lattice with an example.

(g) Write De Morgan's law for quantifiers.

Group-B

10 Marks

Answer any **two** questions : [2×5=10]

2. Prove that the negation of the biconditional "p if and only if q"$(p \leftrightarrow q)$ is equivalent to the exclusive disjunctive form "Either p or q, but not both "$(p \oplus q)$. [5]

3. (a) Let $(S, \leq)$ be a poset. If $a, b \in S$ have a greatest lower bound, show that it is unique. [2]

(b) Prove that there does not exist a Boolean algebra containing only three elements.[3]

4. (a) Let X be a finite set having n elements. Show that $|P(X)| = 2^{|X|}$, where $P(X)$ is the power set of X. [3]

(b) Find the function f of three variables x, y, z such that

$$\begin{aligned} f(x,y,z) &= 1, \text{ if at least two of the variables are } 0 \\ &= 0, \text{ otherwise.} \end{aligned}$$

[2]

5. (a) An integer m is said to be related to another n if m is a multiple of n. Check if the relation is reflexive, symmetric and transitive. [3]

(b) Prove that the complement of each element is unique in a Boolean algebra B. [2]

Group-C

18 Marks

Answer any **two** questions : [2×9=18]

6. (a) Show that in a Boolean algebra $(B, +, \cdot, ')$, $(a \cdot b) + (a' \cdot b) + (a \cdot b') + (a' \cdot b') = I$, for all $a, b \in B$. [3]

(b) A Boolean function f is defined by $f(x,y,z) = (xy+xz')' + y'$. Find the disjunctive normal form of $f(x,y,z)$. [3]

(c) Define free and bound variables. Show by an example that the union of two transitive relations on a set X is not a transitive relation on X. [1+2]

7. (a) Define partition on a set. Prove that an equivalence relation determines a partition. [1+5]

(b) If $A = \{x : 2\cos^2 x + \sin x \leq 2\}$ and $B = \{x : \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}\}$, then find $A \cap B$. [3]

8. (a) Let A, B be subsets of a universal set. Prove that $A = B$ if and only if $A \triangle B = \emptyset$. [5]

(b) Find the number of different reflexive relations on a set containing n elements. [4]