SEM-IV/H/23(CBCS)/Final(GM)

2023

MATHEMATICS (Honours)

Paper Code: DC-H-08

[New Syllabus]

Full Marks: 32 **Time: Two Hour**

Notations and symbols have their usual meanings

Group-A
(Marks 4)

1. Answer any four questions $1 \times 4 = 4$

   (a) Find the differential equation of all circles in xy-plane. [1]

   (b) State Picard's theorem regarding the existence and uniqueness of the solution of the differential equation $\frac{dy}{dx} = f(x, y)$ [1]

   (c) Show that $P_n(-x) = (-1)^n P_n(x)$ [1]

   (d) Prove that $J_{-1/2}(x) = (-1)^n \sqrt{2/\pi x} \cos x$ [1]

   (e) Define Strum-Liouville problem. Give an example of it. [1]

   (f) Give geometrical interpretation of Complete integral and Singular integral of $f(x, y, z, p, q) = 0$ [1]

   (g) Find the curve for the length of the perpendicular from the pole of the tangent varies as the radius vector. [1]

Group-B
(Marks 10)

Answer any two questions: $2 \times 5 = 10$

2. Reduce the differential equation $(px^2 + y^2)(px + y) = (p + 1)^2$ to Clairaut's form then obtain its complete primitive and singular solution if any. [5]

3. Find the general power series solution near $x = 0$ of the equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + p(p+1)y = 0$$

where p is an arbitrary constant. [5]

4. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = \frac{e^x}{1+e^x}$$

[5]

5. Solve the partial differential equation

$$xp + yq = z - a\sqrt{x^2+y^2+z^2}$$

[5]

Group-C

(Marks 18)

Answer any two questions. $2 \times 9 = 18$

6. (a) Find the eigen-values and eigen vectors of

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + (1+\lambda)y = 0$$

satisfying the boundary conditions $y(0) = 0$ and $y'(a) = 0$ [5]

(b) Show that the solution of the equation $\frac{dy}{dx} = Q + Py$ is of the form

$$y = e^{\int P dx}\left[\int e^{-\int P dx} d\left(\frac{Q}{P}\right) - C\right] - \frac{Q}{P}.$$

[4]

7. (a) Solve by using Charpit's method of the equation

$$(p+q)(z - px - qy) = 1$$

[4]

(b) Show that $P_n(x)$ is the coefficient of z^n in the expansion of $(1 - 2xz + z^2)^{-1/2}$ in ascending power of z. [5]

8. (a) Find the integral surface of the linear first order partial differential equation $yp + xq = z - 1$, which passes through the curve $z = x^2 + y^2 + 1, y = 2x$. [5]

(b) Solve the second order differential equation

$$\frac{d^2y}{dx^2} - (1+4e^x)\frac{dy}{dx} + 3e^{2x}y = e^{2(x+e^x)}$$

. [4]

Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: DC09

Semister IV

Mechanics

Full Marks : 32 Time : two hours

(GROUP A)

(4 Marks)

1. Answer any four. 4×1=4

   (a) Write Kepler's law of planetary motion.

   (b) Show that at an apse p=r, where p,r has usual meaning.

   (c) Define static and dynamic friction.

   (d) Show that the work done by a force is equal to the sum of the works done by its components.

   (e) Write the energy test of stability.

   (f) A shell of mass m is ejected from a gun of mass M by an explosion which generates kinetic engergy E. Prove that the initial velocity of the shell is $\sqrt{\frac{2ME}{(M+m)m}}$

   (g) Discuss stability of orbits.

(GROUP B)

(10 Marks)

Answer any two. 2×5=10

2. Show that, if a system of coplanar forces acting at different points of a body have a single resultant and if each force be turned in the plane of the forces about its point of application through the same angle in the same sence, their resultant will always pass through a fixed point. [5]

3. A solid hemisphere is supported by a string fixed to point on its rim and to a point on a smooth vertical wall with which the curved surface of the sphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$ [5]

4. A particle is describing a circle of radius a in such a way that its tangential acceleration is k times the normal accelaration, where k is a constant. If the speed of the particle at any point be u, prove that it will return to the same point after a time

$$\frac{a}{ku}(1-e^{-2\pi k})$$

[5]

5. If a planet was suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. [5]

(GROUP C)
(18 Marks)

Answer any two. 2×9=18

6. (a) Two equal forces act along each of the straight lines

$$\frac{x \mp a\cos\theta}{a\sin\theta} = \frac{y - b\sin\theta}{\mp b\cos\theta} = \frac{z}{c}$$

show that their central axis must, for all values of θ, lie on the surface

$$y(\frac{x}{z}+\frac{z}{x}) = b(\frac{a}{c}+\frac{c}{a})$$

[5]

(b) A solid homogenious hemisphere rests on a rough horizontal plane and against a rough vertical wall, the coefficients of friction being μ and $\mu\ prime$ respectively. Show that the least angle that the base of the hemisphere can make with the vertical is

$$\cos^{-1}(\frac{8\mu}{3}\frac{1+\mu}{1+\mu\mu\prime})$$

[4]

7. (a) A solid frustum of paraboloid of revolution of height h and latus rectum 4a, rests, with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is 4b. Show that equilibrium is stable if $h < \frac{3ab}{a+b}$ [4]

(b) A heavy uniform chain of length 2l, hangs over a small smooth fixed pulley, the length l+c being at one edge and l-c at the other. If the end of the shorter portion be held and then let go, show by the principle of energy, that the chain will slip off the pulley in time

$$\sqrt{\frac{l}{g}}log_e\frac{l+\sqrt{l^2-c^2}}{c}d$$

[5]

8. (a) If the orbit described by a particle under a central force to the origin be $r^n \cos n\theta = a^n$, find the law of force. [5]

(b) A particle of mass m is acted on by a force $m\mu(x+\frac{a^4}{x^3})$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$ [4]

***

GOUR MAHAVIDYALAYA

MATHEMATICS (Honours)

Paper Code: MATH-DC10

Semester-IV

Internal Examination

Time : 2 hour Full Marks : 32

Group-A

1. Answer any **four** questions. 4

(a) A coin is tossed and a die is thrown simultaneously. Write down the sample space.

(b) Define Certain and impossible Events.

(c) Prove that $E(aXb) = aE(X) + b$, where a and b are constants.

(d) A random variable X has the pdf

$$f(x) = \begin{cases} \frac{1}{4}, -2 < x < 2 \\ 0, \ otherwise, \end{cases}$$

find $P(X < 1)$.

(e) Find the mean and median of 20, 15, 25, 7, 25, 17, 5.

(f) What do you mean by 'Parameter' and 'Statistic'? Explain about 'Type-II error' in statistical hypotheses

P.T.O.

Group-B

2. Answer any **two** questions. 2× 5=10

(a) (i) The probability that a particular sum will be solved by A and B are 0.5 and 0.6 respectively. Find the probability that the sum will be solved.

(ii) Show that the second order moment of a random variable X is minimum when taken about its mean. [2+3]

(b) A random variable X has a density function $12x^2(1-x), (0 < x < 1)$. Compute $P(|X - m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality. [5]

3. If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} 8xy, 0 < x < y, 0 < y < 1 \\ 0, elsewhere, \end{cases}$$

Examine whether X and Y are independent. Also compute $Var(X)$ and $Var(Y)$. [5]

4. (i) A coin is tossed until a head appears. What is the expectation of the number of tosses required?

(ii) Find the moment generating function for Binomial Distribution. Hence find its mean and variance. [2+3]

P.T.O.

Group-C

Answer any **two** questions. 2× 9=18

5. (a) Show that the sample variance is a biased estimator of the population variance. [4]

   (b) Find the maximum likelihood estimates of the parameters m and σ in Normal (m, σ). [5]

6. (a) If X and Y are standardized random variables and $r(aX + bY, bX + aY) = \frac{1+2ab}{a^2+b^2}$, find $r(X, Y)$, the correlation coefficient between X and Y. [5]

   (b) If the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), 0 < x < 1, 0 < y < 1 \\ 0, elsewhere, \end{cases}$$

   find the conditional mean and conditional variance of X and $Y = \frac{1}{2}$. [2+2]

7. (a) In a random sample of 400 articles 40 are found to be defective. Obtain 95% confidence interval for the true proportion of defectives in the population of such articles.[Given $P(X > 1.96) = 0.025$]. [4]

   (b) A bird watcher sitting in a park has spotted a number of birds belonging to 6 categories. The exact classification is given below

P.T.O.

Category:	1	2	3	4	5	6
Frequency:	6	7	13	17	6	5

Test at 5% level of significance whether or not the data is compatible with the assumption that this particular park is visited by birds belonging to these six categories in the proportion 1:1:2:3:1:1. [5]

❄