SEM-II/G/23(CBCS)

GOUR MAHAVIDYALAYA

MATHEMATICS (General)

Paper Code: MATH-DC-02/GE-02(Internal)

Full Marks: 32 Time: Two Hour

Group-A

1. Answer any four questions. $1 \times 4 = 4$

   (a) What do you mean by a absolutely convergence series?

   (b) Solve the differential equations $\frac{dy}{dx} = \sin(x+y)$

   (c) Find the value of $B(\frac{1}{2}, \frac{5}{2})$, where B denotes the beta function.

   (d) Find the limit $\lim_{n\to\infty}(2^n + 3^n)^{1/n}$

   (e) Test the differentiability of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x|x|$ for all $x \in \mathbb{R}$, at the point $x = 0$.

   (f) Find the integrating factor of the differential equation $(x^3 + xy^4)dx + 2y^3dy = 0$

Group-B

Answer any two questions. $5 \times 2 = 10$

2. State Cauchy's general principal of Convergence of a real sequence. Use Cauchy's general principal of convergence to prove that the sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, is not convergence. [2+3=5]

3. Solve the differential equation $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$ [5]

4. Examine the convergence of the improper integral $\int_0^\infty \frac{dx}{x^{\frac{1}{2}}(1+x^{\frac{1}{4}})}$ [5]

5. If $I_n = \int \frac{\sin nx}{\sin x}dx$, show that $(n-1)\{I_n - I_{n-2}\} = 2\sin(n-1)x$ [5]

Group-C

Answer any two questions. $2 \times 9 = 18$

6. (a) Test the convergence of the series $1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{1\cdot3}{2\cdot4}\cdot\frac{1}{5} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{7} + \cdots\cdots$ [5]

   (b) Evaluate $\lim_{x\to0^+}(\frac{1}{x} - \frac{1}{\sin x})$ [4]

7. (a) State Rolle's theorem.Verify Rolle's theorem of the function $f(x) = x^2 + \cos x$ on $[-\frac{\pi}{4}, \frac{\pi}{4}]$. [5]

   (b) If $y = x^{n-1}\log x$,$n \in \mathbb{N}$, then prove that $y_n = \frac{(n-1)!}{x}$ [4]

8. (a) Find the value of $\int_0^{\pi/2} \sin^5\theta \cos^7\theta d\theta$, [5]

(b) Find integrating factor of the differential equation $(xy^2 - e^{1/x^3})dx - x^2ydy = 0$, then solve it. [4]
