

2020

MATHEMATICS (Honours)

Paper Code : I - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

Subject Code :

III	A	&	B
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Subject Name :

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- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

Example — If alternative A of 1 is correct, then write :

1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code :

III	A	&	B
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Subject Name :

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A)/(B)/(C)/(D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি 1 নম্বর প্রশ্নের সঠিক উত্তর A হয় তবে লিখতে হবে :

1. – A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : I - A

Full Marks : 20

Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. If the equation $x^n - nqx + (n - 1)r = 0$ has a pair of equal roots, then

- A. $r^{n-1} = q^n$
- B. $q^n r = 1$
- C. $q^n + 1 = r^2$
- D. $r^n - 1 = q^2$.

2. If $a, b, c > 0$ and $a + b + c = 1$, then $(1 + a)(1 + b)(1 + c)$ has

- A. maximum value 8
- B. minimum value 8
- C. maximum value $\frac{64}{27}$
- D. minimum value $\frac{64}{27}$.

3. The unit digit of 7^{100} is

- A. 1
- B. 3
- C. 5
- D. 7.

4. If $x + \frac{1}{x} = 2 \cos \frac{\pi}{7}$, then the value of $x^7 + \frac{1}{x^7}$ is

- A. -1
- B. 1
- C. 2
- D. -2.

5. $\begin{vmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{vmatrix}$ is equal to

- A. $(z-y)(z-x)(y-x)$
- B. $(x-y)(x-z)(y-z)$
- C. $(x-y)^2(y-z)^2(z-x)^2$
- D. $(x^2-y^2)(y^2-z^2)(z^2-x^2)$.

6. The number of different reflexive relations of a set of four elements is

- A. 2^2
- B. 2^4
- C. 2^9
- D. 2^{12} .

7. Which of the following matrices has the same row space as the matrix $\begin{bmatrix} 4 & 8 & 4 \\ 3 & 6 & 1 \\ 2 & 4 & 0 \end{bmatrix}$?

- A. $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- B. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- C. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

8. If for any two vectors α, β of an Euclidean space we have $\|\alpha + \beta\| = \|\alpha - \beta\|$, then
- A. $\langle \alpha, \beta \rangle = 0$
 - B. $\langle \alpha, \beta \rangle = 1$
 - C. $\|\alpha\| = \|\beta\|$
 - D. None of the above.
9. If R is a ring such that $a^2 = a$ for all $a \in R$, then we have
- A. R is not commutative
 - B. R is commutative
 - C. $a = 0$ for all $a \in R$
 - D. None of the above.
10. The total number of unit elements in the ring $(\mathbb{Z}_{10}, +, \cdot)$ is
- A. 4
 - B. 0
 - C. 5
 - D. 9.
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2020

MATHEMATICS (Honours)**Paper Code : I - B****[New Syllabus]**

Full Marks : 80

Time : Three Hours Thirty Minutes

*The figures in the margin indicate full marks.**Notations and symbols have their usual meanings.***Group-A
(30 Marks)**Answer any **six** questions

5 × 6 = 30

1. If a_1, a_2, \dots, a_n are n positive rational numbers and $S = a_1 + a_2 + \dots + a_n$, then prove that $\left(\frac{S}{a_1} - 1\right)^{a_1} \left(\frac{S}{a_2} - 1\right)^{a_2} \dots \left(\frac{S}{a_n} - 1\right)^{a_n} \leq (n-1)^S$. [5]
2. If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$, then prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = \frac{3}{2}$. [5]
3. Prove that $x^n - 1 = (x^2 - 1) \prod_{k=1}^{\frac{n-2}{2}} \left[x^2 - 2x \cos \frac{2k\pi}{n} + 1 \right]$, if n is an even positive integer. Deduce that $\sin \frac{\pi}{32} \sin \frac{2\pi}{32} \dots \sin \frac{15\pi}{32} = \frac{1}{2^{13}}$. [5]
4. If $\log \sin (\theta + i\phi) = \alpha + i\beta$ where $\theta, \phi, \alpha, \beta$ are real, then prove that
 - (a) $2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$ [2]
 - (b) $\cos (\theta - \beta) = e^{2\phi} \cos (\theta + \beta)$. [3]
5. If p is a prime number, then prove that $(p-1)! + 1 \equiv 0 \pmod{p}$. [5]
6. Prove that for any positive integer n , $4 \cdot 6^n + 5^{n+1} - 9$ is divisible by 2. [5]

7. If a, b, c and d are roots of the equation $x^4 + x^3 + 2x^2 + x + 1 = 0$, then find the value of

(a) $(a^2 + 4)(b^2 + 4)(c^2 + 4)(d^2 + 4)$ [3]

(b) $(a^3 + 1)(b^3 + 1)(c^3 + 1)(d^3 + 1)$. [2]

8. Find the special roots of the equation $x^{20} - 1 = 0$. Deduce that $\cos \frac{\pi}{10}, \cos \frac{3\pi}{10}, \cos \frac{7\pi}{10}$ and $\cos \frac{9\pi}{10}$ are roots of the equation $16y^4 - 20y^2 + 5 = 0$. [5]

9. If α, β, γ are roots of the equation $x^3 - 2x^2 + 3x - 1 = 0$, then find the equation whose roots are $\frac{\beta\gamma - \alpha^2}{\beta + \gamma - 2\alpha}, \frac{\gamma\alpha - \beta^2}{\gamma + \alpha - 2\beta}, \frac{\alpha\beta - \gamma^2}{\alpha + \beta - 2\gamma}$. [5]

Group-B
(25 Marks)

Answer any five questions

$5 \times 5 = 25$

10. Prove that every permutation on a finite set is either a cycle or it can be expressed as a product of disjoint cycles. [5]

11. Show that the intersection of two subgroups of a group G is a subgroup of G . Is the result true in case of union? Justify your answer. [5]

12. Show that the order of a cyclic group is equal to the order of its generator. [5]

13. Prove that a non-trivial finite ring having no divisors of zero is a ring with unity. Can it be extended to an integral domain? Explain with an example. [5]

14. Define a subfield of a field. Prove that $\mathbb{Q}[\sqrt{2}]$ forms a subfield of \mathbb{R} . [5]

15. If $f : A \rightarrow B$ is a mapping and P, Q are nonempty subsets of A , then show that $f(P \cup Q) = f(P) \cup f(Q)$. [5]

16. Prove that every field is an integral domain. Is the converse true? Justify your answer with an example. [5]

Group-C
(25 Marks)

Answer any **five** questions

$5 \times 5 = 25$

17. If $2s = a + b + c$, then show that
$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c).$$
 [5]

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence evaluate A^{50} . [5]

19. Find the value of k , if the following system:

$$\begin{aligned} x - y + z &= 2 \\ 2x + 3y - 4z &= k \\ 3x + 4y + 5z &= k^2 \end{aligned}$$

has

- (i) unique solution
- (ii) infinitely many solutions
- (iii) no solution. [5]

20. Use the Gram-Schmidt method of orthogonalization to construct an orthonormal basis for the subspace of \mathbb{R}^4 generated by $(1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, 2)$. [5]

21. Prove that row rank and column rank of a matrix are equal. [5]

22. Let $A = (a_{ij})_{n \times n}$ be a square matrix and A_{ij} be the co-factor of a_{ij} in $\det A$. Prove that $\det(A_{ij}) = (\det A)^{n-1}$. [5]

23. Let A be a 3×3 real matrix and $(1, 2, 1)$, $(0, 1, 1)$, $(1, 1, 1)$ be eigenvectors corresponding to the eigenvalues $2, 3, 1$ respectively. Find the matrix A . [5]
24. Let S denote the set of four vectors $(1, 2, -1)$, $(-3, -6, 3)$, $(2, 1, 3)$, $(8, 7, 7)$. Find a linearly independent subset of S which spans the same space as S . [5]
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