2020

MATHEMATICS (Honours)

Paper Code : IV - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : IV - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. The set $X = \{(x_1, x_2) \mid x_1 + 2x_2 = 5\}$ is

   A. a hyperplane

   B. a polyhedron

   C. a convex set

   D. none of the above.

2. For the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 - x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + 4x_3 - x_4 &= 0 \end{aligned}$$

   the solution $(1, 0, 1, 6)$ is

   A. basic and feasible

   B. not basic but feasible

   C. basic and degenerate

   D. basic and non-degenerate.

3. The value of the game

	PLAYER A	
PLAYER B	6	-4
	-1	2

   is

   A. $\frac{8}{13}$

   B. $\frac{7}{13}$

   C. $\frac{5}{13}$

   D. $\frac{6}{13}$.

4. If $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are three vectors such that $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\vec{0}$ and $|\vec{\alpha}|=3$, $|\vec{\beta}|=5$, $|\vec{\gamma}|=7$, then the angle between $\vec{\alpha}$ and $\vec{\beta}$ is

   A. $\frac{\pi}{6}$
   B. $\frac{\pi}{2}$
   C. $\frac{\pi}{3}$
   D. $\frac{\pi}{4}$.

5. The solution of the PDE $a(p+q)=z$ is

   A. $\phi(x+y, y-az)=0$
   B. $\phi(x+y, y+az)=0$
   C. $\phi(x-y, y+az)=0$
   D. $\phi(x-y, y-az)=0$.

6. The L.P.P.

$$\begin{aligned} \text{Maximize} \quad & z = 7x_1 + 5x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 6, \\ & 4x_1 + 3x_2 \leq 12, \\ & x_1, x_2 \geq 0 \end{aligned}$$

   has the solution

   A. $x_1=2, x_2=0; z_{\max}=14$
   B. $x_1=3, x_2=0; z_{\max}=21$
   C. $x_1=1, x_2=2; z_{\max}=17$
   D. $x_1=3, x_2=1; z_{\max}=26$.

7. The value of λ for which the vectors $\lambda\hat{i}-4\hat{j}+5\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$ are coplanar, is

   A. $\frac{28}{3}$
   B. $\frac{3}{29}$
   C. $\frac{29}{3}$
   D. $\frac{3}{28}$.

8. The work done by the force $2\widehat{i} + 4\widehat{j} - \widehat{k}$ when it produces a displacement from the point $\widehat{i} + \widehat{j} + 2\widehat{k}$ to $3\widehat{i} - \widehat{j} - \widehat{k}$ is

A. -1

B. 1

C. 2

D. 3.

9. An integrating factor of the differential equation $(x^2 + y^2 + x)dx + xy\,dy = 0$ is

A. $\frac{1}{x}$

B. x

C. x^2

D. $\frac{1}{x^2}$.

10. The initial solution of the following transportation problem using matrix minima method will be

		D_1	D_2	D_3	D_4	Supply
		Destination				
Source	O_1	1	2	1	4	30
	O_2	3	3	2	1	50
	O_3	4	2	5	9	20
	Demand	20	40	30	10	

A. $x_{11} = 20, x_{12} = 10, x_{22} = 30, x_{23} = 20, x_{33} = 10, x_{34} = 10$; total cost = 310

B. $x_{11} = 20, x_{13} = 10, x_{22} = 20, x_{23} = 20, x_{24} = 10, x_{32} = 20$; total cost = 180

C. $x_{11} = 20, x_{12} = 10, x_{22} = 30, x_{23} = 20, x_{32} = 10, x_{34} = 10$; total cost = 310

D. $x_{11} = 20, x_{12} = 10, x_{21} = 30, x_{24} = 20, x_{33} = 10, x_{34} = 10$; total cost = 180.

2020

MATHEMATICS (Honours)

Paper Code : IV - B

[New Syllabus]

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A
(20 Marks)

Answer any **four** questions

$5 \times 4 = 20$

1. Prove by vector method, that $\sin(A-B) = \sin A\cos B - \cos A\sin B$.

2. Show that $\left[\vec{\beta}\times\vec{\gamma}, \vec{\gamma}\times\vec{\alpha}, \vec{\alpha}\times\vec{\beta}\right] = \left[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}\right]^2$.

3. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three non-coplanar vectors, then show that for any vector $\vec{r}$,

$$\vec{r} = \frac{\left[\vec{r}\,\vec{b}\,\vec{c}\right]}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}\vec{a} + \frac{\left[\vec{a}\,\vec{r}\,\vec{c}\right]}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}\vec{b} + \frac{\left[\vec{a}\,\vec{b}\,\vec{r}\right]}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}\vec{c}.$$

4. If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$, then show that $\left|\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}\right|^2 = a^2(a^2+b^2)$ and

$$\frac{d\vec{r}}{dt}\cdot\left(\frac{d^2\vec{r}}{dt^2}\times\frac{d^3\vec{r}}{dt^3}\right) = a^2b.$$

5. If $f(r)$ is differentiable, then prove that the vector $f(r)\vec{r}$ is irrotational.

6. If the volume of a tetrahedron is 2 cubic unit and three of its vertices are $(1,0,1)$, $(1,1,0)$, $(2,-1,1)$, find the locus of its fourth vertex.

Group-B
(30 Marks)
Answer any **six** questions

$5 \times 6 = 30$

7. Reduce the equation
$$xyp^2 - (x^2 + y^2 - 1)p + xy = 0$$
to Clairaut's form by the substitution $x^2 = u$ and $y^2 = v$. Hence show that the equation represents a family of conics touching the four sides of a square.

8. Find the orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter.

9. Using Lagrange's method, solve the PDE: $z(x + y)p + z(x - y)q = x^2 + y^2$.

10. Solve:
$$\frac{dx}{dt} + 5x + y = e^t, \ \frac{dy}{dt} - x + 3y = e^{2t}.$$

11. If $y = x$ is a solution of $x^2y'' + xy' - y = 0$, then find the solution.

12. Using the method of variation of parameters, solve the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$.

13. Find a complete integral of $p^2 - y^2q = y^2 - x^2$ by Charpit's method.

14. Solve: $xy'' - (2x - 1)y' + (x - 1)y = 0$.

15. Find the eigenvalues λ_n and eigenfunctions $y_n(x)$ for the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \lambda y = 0, (\lambda > 0)$ satisfying the boundary conditions $y'(1) = 0 = y'(e^{\pi/2})$.

Group-C
(30 Marks)
Answer any **six** questions

$5 \times 6 = 30$

16. If x^0 is any feasible solution to the primal problem and v^0 is any feasible solution to the dual problem, show that $cx^0 \leq b^T v^0$.

17. Use two-phase method to solve the LPP:

$$\begin{aligned}
\text{Minimize} \quad & z = x_1 + x_2 + x_3 \\
\text{subject to} \quad & x_1 - 3x_2 + 4x_3 = 5, \\
& x_1 - 2x_2 \leq 3, \\
& 2x_2 + x_3 \geq 4, \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}$$

18. Solve the following LPP by simplex method:

$$\begin{aligned} \text{Minimize} \quad & z = x_1 - 3x_2 + 2x_3 \\ \text{subject to} \quad & 3x_1 - x_2 + 2x_3 \le 7, \\ & -2x_1 + 4x_2 \le 12, \\ & -4x_1 + 3x_2 + 8x_3 \le 10, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

19. A company has four machines on which to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the adjacent tableau. What are the job assignment which will minimize the cost?

		MACHINE			
		W	X	Y	Z
	A	18	24	28	32
JOB	B	8	13	17	19
	C	10	15	19	22

20. Solve the following transportation problem:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

21. Solve the following game by linear programming technique:

	PLAYER B		
	1	-1	3
PLAYER A	3	5	-3
	6	2	-2

22. Use dual simplex method to solve

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 - x_2 + x_3 \ge 4, \\ & x_1 + x_2 + 2x_3 \le 8, \\ & x_2 - x_3 \ge 2, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

23. If for any basic feasible solution x_B of an LPP, at any iteration of simplex algorithm, $z_j - c_j \geq 0$ for all non-basic vectors, then show that x_B is an optimal solution.

24. Use graphical method in solving the following game:

	PLAYER B	
PLAYER A	2	4
	2	3
	3	2
	-2	6