2020

MATHEMATICS (Honours)

Paper : MTMH-DC-4

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Let G be a group and $a, b \in G$ be such that $a^4 = e$ and $ab = ba^2$. Prove that a=e.

(b) Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 4 & 1 & 2 & 6 & 8 & 7 \end{pmatrix}$ as the product of disjoint cycles.

(c) Show that $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}^+, .)$.

(d) Show that in a non-trivial ring R with unity, zero element has no multiplicative inverse.

(e) If a is a fixed element of a ring R, then prove that the set $S = \{x \in R : xa = 0\}$ is a subring of R.

(f) Let R be a ring with unity $1 \neq 0$ such that R has no nontrivial proper left ideal. Show that R is a division ring.

(g) Define maximal ideal.

Group - B

(10 Marks)

Answer any *two* questions. $5 \times 2 = 10$

2. Show that every permutation can be written as the product of disjoint cycles. 5

3. Find all the homomorphisms of the group $(\mathbb{Z}, +)$ to the group $(\mathbb{Z}, +)$. 5

4. State and prove the first isomorphism theorem of rings. 5

5. Find all prime ideals and maximal ideals in the ring $\mathbb{Z}_8$. 5

Group - C

(18 Marks)

Answer any *two* questions. $9 \times 2 = 18$

6. (a) Show that a subgroup H of a group G is normal if and only if $aHa^{-1} = H$, for all $a \in G$. 5

   (b) Consider the groups $G = (\mathbb{R}, +)$ and $H = (\mathbb{Z}, +)$. Let S be the subgroup of all nonzero complex numbers with unit modulus. Prove that the quotient group G/H is isomorphic to S. 4

7. (a) Let R and S be two rings and $f : R \to S$ be a ring homomorphism. Show that $\ker f$ is an ideal of R. 4

   (b) Let R be a commutative ring with unity. Then show that every maximal ideal of R is a prime ideal. 3

   (c) Show that the ideal $I = \langle 4 \rangle$ (generated by 4) of $2\mathbb{Z}$ is maximal but not prime. 2

8. (a) Let R be a commutative ring with unity $1 \neq 0$. Then show that an ideal M of R is maximal if and only if R/M is a field. 5

(b) With respect to usual addition and multiplication of matrices, show that the ring $\left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{Q} \right\}$ forms a field but the ring

$$\left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

does not form a field. 4