2020

MATHEMATICS (Honours)

Paper : MTMH - DC-01

[CBCS]

Full Marks : 32   Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions. $1 \times 4 = 4$

   (a) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x|x|$ is differentiable at every point.

   (b) Show that $\lim_{x \to 0} \cos \frac{1}{x}$ does not exist.

   (c) State Leibnitz rule of successive differentiation.

   (d) Obtain an equation of second degree that represents a pair of straight lines passing through the origin.

   (e) How many normals can be drawn from a given point to a parabola?

   (f) Obtain the radius of the circle of intersection of a sphere of radius 13 cm by a plane at a distance 5 cm from the centre of the sphere.

   (g) Write down the name of the quadric surface represented by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$.

Group - B

Answer any *two* questions. 5×2=10

2. If $\log y = \tan^{-1} x$, then prove that $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}+n(n+1)y_n = 0$. [5]

3. Find the envelope of circles whose centre lie on the rectangular hyperbola $xy = c^2$ and passes through the origin. [5]

4. Prove that the chord of contact of two mutually perpendicular tangents drawn from a point to a parabola passes through its focus. [5]

5. If the guiding curve of a right circular cylinder is the circle $x^2+y^2+z^2 = 9, x-y+z = 3$, then find the equation of the cylinder. [5]

Group - C

Answer any *two* questions. 9×2=18

6. (a) Use L'Hospital's rule to evaluate
$$\lim_{x \to -4} \frac{\sin(\pi x)}{x^2 - 16}.$$
[2]

(b) Use Taylor's theorem to prove that
$$1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2}, \quad \text{if} \quad x > 0.$$
[3]

(c) Obtain the condition for a general equation of second degree that represents a pair of straight lines. [4]

7. (a) Plot the graph of e^x and use it to plot the graph of the function e^{x+5}. [2]

(b) Find the length of the radius of curvature of the rectangular spiral $r = ae^{\theta \cot \alpha}$ at (r, θ). [3]

(c) Find the locus of the point of intersection of the perpendicular generators of the following hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. [4]

8. (a) If $I_n = \int \sin^n x\, dx$, then show that $I_n = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$. [3]

(b) Find the asymptotes of the curve $(y-2x)^2(y-x)-3(y-2x)(y-x)+2(y-x)+1 = 0$. [4]

(c) Find the equation of the bisectors of the angles between the pair of straight lines represented by $8x^2 + 10xy + 3y^2 + 26x + 16y + 21 = 0$. [2]

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