2020

MATHEMATICS (Honours)

Paper : MTMH - DC-02

[CBCS]

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions $1 \times 4 = 4$

   (a) Find amp(z), where $z = 1 + i\cot\theta$.

   (b) Find the equation whose roots are the roots of the equation $x^3 + 7x + 9 = 0$ each increased by 1.

   (c) Show that the function $f(x) = \frac{x-3}{x+2}$ is one to one on $\mathbb{R} \setminus \{-2\}$.

   (d) Define an equivalence relation on the set of integers.

   (e) State the well ordering property of positive integers.

   (f) Show that the $\gcd(a, a+2) = 1$ or 2 for every integer a.

   (g) State the Cayley-Hamilton theorem.

Group - B

Answer any *two* questions. 5×2=10

2. Prove that $1! \cdot 3! \cdot 5! \cdots (2n-1)! > (n!)^n$. [5]

3. Solve the equation $x^4 + 4x^3 - 6x^2 + 20x + 8 = 0$ by Ferrari's method. [5]

4. (a) Let a and b be two integers. If $a \equiv b \pmod{m}$ for some integer m, then prove that $a^n \equiv b^n \pmod{m}$ for all positive integer n. [2]

   (b) If $a \mid b$ and $a \mid c$, then show that $a \mid (bx + cy)$ for arbitrary integers x and y. [3]

5. Find the values of λ and μ for which the following system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

have
(i) a unique solution and
(ii) an infinite number of solutions. [3+2]

Group - C

Answer any *two* questions. 9×2=18

6. (a) If n is any positive integer > 1, then prove that $\frac{1}{n+1} + \frac{1}{n+3} + \cdots + \frac{1}{3n-1} > \frac{1}{2}$. [5]

   (b) Using Euclidean algorithm, find two integers u and v satisfying $63u + 55v = 1$. [4]

7. (a) Solve the equation $(x+1)^6 = (x-1)^6$. [4]

   (b) Using congruence operation, reduce the quadratic form $2x^2 + y^2 - 3z^2 - 8yz - 4zx + 12xy$ to its normal form and find its rank and signature. [5]

8. (a) Solve the equation $40x^4 - 22x^3 - 21x^2 + 2x + 1 = 0$, given that the roots are in harmonic progression. [4]

(b) Verify Cayley-Hamilton theorem for the matrix

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

and hence evaluate A^{-1}. [3+2]