



**University of Gour Banga**

*Syllabi for*

**Three Years Honours Degree Cours**

(Under 1+1-1 System)

**Mathematics Honours**

**University of Gour Banga**

**P.O. – Mokdumpur,**

**Dist. – Malda**

**West Bengal**

**PIN - 732103**



Congruence relation modulo  $m$  is an Equivalence Relation. Congruence Classes. Mapping: Injection, Surjection, Bijection, Inverse and Identity mapping. Composition of Mappings and its Associativity. Binary operation: Intuitive definition. Definition on the basis of mapping. Binary operation in a finite set by Cayley Tables. (10)

2. Introduction to Group Theory: Groupoid, Semi-group, Quasigroup, Monoid, Group. Definition with both-sided Identity and Inverse. (Examples of finite and infinite groups taken from various branches, e.g. from number system, roots of unity, non-singular real matrices of a fixed order, symmetries of a square, triangle, etc.) Additive group of integers modulo an integer  $m$ , Multiplicative group of integers modulo a prime  $p$ . Klein's 4 Group. Properties deducible from the definition of group including solvability of  $ax = b$  and  $ya = c$ . Any finite semi-group, in which both cancellation laws hold, is a group. Integral powers of an element and laws of indices in a group. Order of a group and order of an element of a group.

Subgroups: Necessary and sufficient condition for a sub-set of a group to be a sub-group. Intersection and Union of two sub-groups. Necessary and sufficient condition for the union of two sub-groups to be a sub-group.

Cosets and Lagrange's Theorem. Cyclic group: Definition and examples. Sub-groups of a cyclic group, Generator. Necessary and sufficient condition for a finite group to be cyclic. Permutations: Cycle, Transposition. Every  $\sigma \in S_n$  (Symbols have their usual meanings) can be expressed as the product of disjoint cycles. Even and odd permutations. Symmetric group. Alternating group: Order of an alternating group. (20)

3. Introduction to Rings and Fields: Ring: Definition and examples. Ring of integers modulo  $n$ . Properties directly following from the definition. Multiplicative identity in a ring. Commutative ring. Divisors of zero. Commutative ring with identity and without zero divisor -Integral Domain.

Field: Definition and examples. Every field is an integral domain. Every finite integral domain is a field. Sub-ring and Sub-field. Necessary and sufficient condition of a sub-set of a ring (a field) to be a sub-ring (sub-field). Characteristic of a ring and of an integral domain. (10)

### Group -C (30 Marks)

#### (Linear Algebra)

1. Matrices of real and complex numbers: Definition of a matrix. Equality of matrices. Addition, multiplication, scalar multiplication. Transpose of a matrix. Symmetric, Skew-symmetric and Hermitian matrix. Orthogonal matrix. (5)

2. Determinants: Definition of a determinant of a square matrix. Basic properties. Minors and Cofactors. Expansion of determinant, Laplace's method. Product of determinants. (10)

**PAPER - I**  
**Group -A (40 Marks)**  
**(Classical Algebra)**

1. Integers: (It is not the aim to give an axiomatic development of the topic, rather assume that the students are familiar with the set  $\mathbb{Z}$  of integers, the elementary properties of addition, multiplication and order)

Statements of well ordering principle, first principle of mathematical induction, second principle of mathematical induction. Proofs of some simple mathematical results by induction. Divisibility of integers. The division algorithm ( $a = qb + r$ ,  $b \neq 0, 0 \leq r < b$ ). The greatest common divisor (g.c.d.) of two integers  $a$  and  $b$ . [This number is denoted by the symbol  $(a,b)$ ]. Existence and uniqueness of  $(a,b)$ . Relatively prime integers. The equation  $ax + by = c$  has integral solution iff  $(a,b)$  divides  $c$ . ( $a,b,c$  are integers).

Prime integers: Euclid's first theorem: If some prime  $p$  divides  $ab$ , then  $p$  divides either  $a$  or  $b$ . Euclid's second theorem: There are infinitely many prime integers. Unique factorisation theorem. Congruences, Linear Congruence's. Statement of Chinese Remainder Theorem and simple problems, Theorems of Fermat. Multiplicative function  $\phi(n)$ . (15)

2. Complex Numbers: De-Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of  $a^x$  ( $a \neq 0$ ). Inverse circular and Hyperbolic functions. (8)

3. Polynomials with real coefficients: Fundamental theorem of classical Algebra (statement only). The  $n$  th degree polynomial equation has exactly  $n$  roots. Nature of roots of an equation (Surd or complex roots occur in pairs). Statements of Descartes' rule of signs and of Sturm's Theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots. Reciprocal equations.

Cardan's method of solving a cubic equation. Ferrari's method of solving a biquadratic equation. Binomial equations. Special roots. (15)

4. Inequalities  $AM \geq GM \geq HM$  and their generalisations: the theorems of weighted means and  $m$  th power theorem. Cauchy's inequality (statement only) and its direct applications. (5)

**Group -B (30 Marks)**  
**(Modern Algebra)**

1. Basic concepts: Sets, Sub-sets, Equality of sets, Operations on sets - Union, Intersection and Complement, Symmetric difference. Properties including De Morgan's laws. Cartesian products: Binary relations from a set to a set domain, range, examples from  $R \times R$ ). Equivalence relation; Fundamental theorem on Equivalence relation (Partition). Relation of Partial

## BOOKS FOR REFERENCE:

1. The Theory of Equations (Vol. I) -Burnside & Panton
2. Higher Algebra -Barnard & Child
3. Higher Algebra. -Kurosh (Mir)
4. Modern Algebra -Surjeet Singh & Zameeruddin
5. First Course in Abstract Algebra -Fraleigh
6. Topics in Algebra -Herstein
7. Linear Algebra -Hadley
8. Text Book of Algebra -Leadership Project committee  
(University of Bombay)
9. Text Book of Matrix -B.S. Vatsa
10. Elements of Abstract Algebra -Sharma, Gokhroo, Saini  
(Jaipur Publishing House,  
S.M.S., Highway, Jaipur-3)
11. Abstract Algebra -N. P. Chaudhuri  
(Tata McGraw Hill)

## Paper II

### Group -A (45 marks) (Real Analysis I)

1. Real numbers: Field axioms for real numbers and other salient properties taken as axioms, Arithmetic continuum, Well-ordering principle for  $\mathbb{N}$ . Concept of ordered field. Concept of point set in one dimension. Bounded set. Least upper bound axiom or Completeness axiom, Archimedean property and density property, characterisation of  $\mathbb{R}$  as a complete, Archimedean, ordered field and  $\mathbb{Q}$  as Archimedean, ordered field. Symbols  $\alpha$  and  $-\alpha$ . Symbols of intervals. (5)

2. Sequence of points in one dimension: Bounds, Limits, Convergence and non-convergence, Operations on limits, Sandwich rule, Monotone sequences and their convergence. Nested interval theorem. Cauchy's general principle of convergence. Cauchy sequence. Limits of some important sequences with special reference to  $\left\{1 + \frac{1}{n}\right\}$ . Cauchy's first and second limit theorems. (10)

Vandermonde's determinant, Solution of the system of equations by Cramer's Rule (Problems of determinants of order greater than 4 will not be asked).  
proof of theorems). (8)

3.

- (a) Adjoint of a square matrix. For a square matrix  $A$  of order  $n$ ,  $A \cdot \text{Adj } A = \text{Adj } A \cdot A = \det A \cdot I_n$ .
- (b) Non-singular matrix iff corresponding determinant is non-zero. Non-singular matrix and Invertible Matrix.
- (c) Elementary operations. Echelon matrix. Rank of a matrix - Determination of rank of a matrix - statement and applications of all relevant results and theorems (No proof required). (5)

4. Normal forms. Elementary matrix: Statement and application of the results on Elementary matrix. The normal form and equivalence of matrices. Congruence of matrices - Statement and application of relevant theorems.

Real Quadratic form involving three variables. Reduction to Normal Form (Statement and application of relevant theorems). (5)

5. Vector/Linear space over a field with special reference to spaces of  $n$  tuples of real numbers. Examples of vector space from different branches of Mathematics. Sub-space. Union and Intersection of vector sub-spaces. Sum of two sub-spaces.

Linear combinations. Linear dependence and independence of a finite set of vectors, Linear span. Generators of a vector space. Finite dimensional vector space.

Existence of Basis, Replacement Theorem. Any two bases have the same number of basis vectors. Extension Theorem - Extraction of basis from generators. Formation of basis from linearly independent sub-set. Special emphasis on  $\mathbb{R}$ . Examples from  $\mathbb{R}^n$  ( $n \leq 4$ ) (10)

6. Row Space and Column Space of a Matrix: Definition of row space and column space of a matrix, Row rank, Column rank, Rank of a matrix.  $\text{Rank}(AB) \leq \min(\text{Rank } A, \text{Rank } B)$ . (6)

7. Linear homogeneous systems of equations: Solution space as a sub-space. For a homogeneous system  $AX = 0$  in  $n$  unknowns,  $\text{Rank } X(A) + \text{Rank } A = n$ . The homogeneous system  $AX = 0$  containing  $n$  equations in  $n$  unknowns has a non-trivial solution iff  $\text{Rank } A < n$ . System of linear non-homogeneous equations: Necessary and sufficient condition for the consistency of the system. Solution of the system of equations (Matrix method, Cramer's Rule). (5)

8. Characteristic equation of a square Matrix. Eigen value and Eigen vector. Cayley-Hamilton Theorem. Simple properties of Eigen value and Eigen vector. Diagonalisation of matrices. (6)

9. Inner Product Space. Definition and examples, Norm, Euclidean vector space (EVS), Triangle inequality and Cauchy - Schwarz inequality in EVS. Orthogonality of vectors, Orthogonal basis, Gram-Schmidt process of orthonormalization. (9)

closed intervals: Boundedness, attainment of bounds, Bolzano's theorem, Intermediate value property and allied results.

Continuous function carries closed and bounded interval into closed and bounded interval. Functions continuous on a closed and bounded interval  $I$  is uniformly continuous on  $I$ . A necessary and sufficient condition under which a continuous function on a bounded open interval  $I$  will be uniformly continuous on  $I$ . Lipschitz condition and uniform continuity. Existence of inverse function of a strictly monotone function and its continuity with special reference to inverse circular functions. (15)

8. Concept of differentiability and differential: Chain rule, sign of derivative. For a differentiable function Lipschitz condition is equivalent to boundedness of the derivative. Successive derivative: Leibnitz theorem. Theorems on derivatives: Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy, Taylor's theorem with Schloerlich-Rouche's form of remainder, Lagrange's and Cauchy's form of remainder. Young's form of Taylor's theorem. Maclaurin's series. Expansion of  $e^x$  ( $x > 0$ ),  $\log(1+x)$ ,  $(1+x)^n$ ,  $\sin x$ ,  $\cos x$  etc. with their ranges of validity. (10)

9. Indeterminate forms: Statement of L. Hospital's rule and its consequences. (2)

10. Point of local extremum (maximum, minimum and saddle point) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point. Applications of the principle of maximum/minimum in geometrical and physical problems.

### Group B (25 Mark.)

#### (Functions of Several Variables)

1. Point set in two and three dimensions -Concept only of neighbourhood of a point, interior point, accumulation point, open set, closed set, Bolzano-Weierstrass theorem (Statement only) in  $R^n$ . (7)

2. Concept (only) of  $R^n$  and examples of functions in  $R^n$ . (3)

3. (a) Functions of two and three variables -Limit and continuity, Partial derivatives. Sufficient condition for continuity. Relevant results regarding repeated limits and double limits.

(b) Functions  $R^2 \rightarrow R^1$  -Differentiability and its sufficient condition, differential as a map, Chain rule, Euler's theorem and its converse. Commutativity of the order of partial derivatives -Theorems of Young and Schwarz. (12)

3. Point set in one dimension: (a) Denumerable, at most denumerable and non-denumerable sets. A sub-set of a denumerable set is either finite or denumerable. Union of (i) a finite set and a denumerable set (ii) two denumerable sets (iii) denumerable number of denumerable set.

Denumerability of rational number. Non-denumerability of points in a finite interval and of the set of all real numbers.

(b) Neighbourhood of a point, Interior point, Accumulation point and Isolated point of a linear point set, Bolzano- Weierstrass Theorem on accumulation point, Derived set, Open set and closed set. Union, Intersection, Complement of open and closed sets in R. No non-empty proper sub-set of R is both open and closed in R. Closure of a set to be defined as the union of the set and its derived set. Interior of a set. Deduction of basic properties of interior of a set and closure of a set.

(10)

4. Sub-sequences: All the sub-sequences of a convergent sequence are convergent and converge to the same limit as that of original sequence. Every bounded sequence has a convergent sub-sequence. Subsequential limits. Upper limit and Lower limit as the L.H.D. and G.L.D. respectively of a set containing all the subsequential limits -other equivalent definitions: Inequalities and equalities with upper and lower limits. A sequence is convergent iff its upper and lower limits be equal.

(10)

5. Infinite series of real numbers: Convergence, divergence, Cauchy's criterion of convergence. Abel-Pringsheim's Test. Series of non-negative real numbers: Tests of convergence -Cauchy's condensation test. Upper limit and lower limit criteria for (i) Comparison test, (ii) Ratio test (iii) Root test, (iv) Kummer's test. Statements of Raabe's test, Bertrand's test, Logarithmic test and Gauss's test.

Series of arbitrary terms: Absolutely convergent and conditionally convergent series. Alternating series: Leibnitz test, Root test and Ratio test. Non-absolute convergence -Abel's and Dirichlet's test (Statements and applications). Rearrangement of series through examples. Riemann's re-arrangement theorem (Statement) and simple examples. Rearrangement of absolutely convergent series.

(10)

6. Real valued functions defined on intervals: Bounded functions, Step functions, Monotone functions, Composition of functions. Limits of functions: Algebra of limits and Sandwich rule. Cauchy criterion for the existence of finite limit. Important limit like  $(\sin x)/x$ ,  $\{\log(1+x)\}/x$ ,  $(a^x - 1)/(a > 0)$  as  $x \rightarrow 0$ . Upper and Lower limit of function at a point.

7. Continuity of a function at a point and on an interval. Neighbourhood properties, continuity of  $x^n$ ,  $\sin x$ ,  $\cos x$ ,  $\log x$  to be established.

Continuity of composite function. Piecewise continuous functions. Uniform continuity. Discontinuity of functions -type of discontinuity, ordinary discontinuity of monotonic functions. Properties of continuous functions on



4. Jacobian for functions of two and three variables. Simple properties including functional dependence. Concept of Implicit function: Statement and simple application of Implicit function theorem for two variables. Differentiation of Implicit function. Jacobian of Implicit function. Partial derivative as ratio of two Jacobians in case of function of two variables. (10)

**Group -C (30 Marks)**  
**(Applications)**

1. Concept of a plane curve: Closed curve, simple curve.

(a) Tangents and Normals: Subtangent and sub-normals. Angle of intersection of curves. Pedal equation of a curve, Pedal of a curve.

(b) Rectilinear asymptotes of a curve (Cartesian, parametric and polar form).

(c) Curvature -Radius of curvature, Centre of curvature, Circle of curvature Evolute of a curve.

(d) Concavity, convexity, singular points, nodes, cusps, points of inflexion -simple problems on species of cusps of a curve.

(e) Envelopes of one parameter and two parameter family of curves. Envelope as singular point locus -Evolute.

(f) Curve tracing -familiarity with well-known curves. (15)

2. Indefinite and suitable corresponding definite integrals for the functions  $1/(a + b \cos x)^n$ ,  $(l \cos x + m \sin x)/(p \cos x + q \sin x)$ ,  $1/(r + a^2)^n$ ,  $\cos^n x \sin nx$  etc. where  $l, m, p, q, n$  are integers. Simple problems on definite integrals as the limit of a sum. (5)

3. Working knowledge of Beta and Gamma function (convergence to be assumed) and their interrelation (no proof). Use of the result  $\Gamma(r)\Gamma(1-r) = \pi / \sin \pi r$  where  $0 < r < 1$ . Computation of the integrals

$$\int_0^{\pi/2} \sin^n x \, dx, \int_0^{\pi/2} \cos^n x \, dx, \int_0^{\pi/4} \tan^n x \, dx \text{ etc. when they exist, (using Beta function and Gamma function). (5)}$$

4. Area: Area enclosed by a curve, area enclosed between a curve and a secant, area between two curves and area between a curve and its asymptote (if there be any). (2)

5. Problems on volume and surface area of solids of revolution. Statement of Pappus theorem and its direct application for well-known curves. (2)

6. Determination of C.G. and moments & products of Inertia -Simple problems only. (3)

**BOOKS FOR REFERENCE:  
(FOR PAPER II & PAPER V)**

- |  |   |
|--|---|
| 1. Basic Real & Abstract Analysis        | -Randolph J. P. (Academic Press)                      |
| 2. A first course in Real Analysis       | -M.H. Protter & G. B. Morrey (Springer Verlag) (NBHM) |
| 3. Introduction to Real Variable Theory  | -Saxena & Shah (Prentice Hall Publication)            |
| 4. A course of Analysis                  | -Philips  |
| 5. Problems in Mathematical Analysis     | -B. P. Demidovich (Mir Publication)                   |
| 6. --do--                                | Berman (Mir)  |
| 7. Differential & Integral Calculus      | -Courant & John (Vol. I) and (Vol. II)                |
| 8. Calculus of one Variable              | -Maron (CBS Publications)                             |
| 9. Introduction to Real Analysis         | -Bartle & Sherbert (John Wiley & Sons)                |
| 10. Mathematical Analysis                | -Parzynski  |
| 11. Real Analysis                        | -Ravi Prakash & Siri Wasan (Tata Mc Graw Hill)        |
| 12. Mathematical Analysis                | -Shantinarayan  |
| 13. Differential Calculus                | -Shantinarayan (S. Chand & Co.)                       |
| 14. Integral Calculus                    | --do--  |
| 15. Theory and applications of Infinite  | -Dr. K. Knopp Series                                  |
| 16. Theory of Integrals & Fourier Series | -H. S. Carslaw  |
| 17. Advanced Calculus                    | -David Widder (Prentice Hall)                         |
| 18. Metric Spaces                        | -E. T. Copson (Cambridge University Press)            |
| 19. Metric Spaces                        | -P. K. Jain & K. Ahmed (Narosa)                       |
| 20. Complex Variables and applications.  | -R. V. Churchill & J. W. Brown (Mc Graw Hill)         |
| 21. Foundations of Complex Analysis      | -S. Ponnusamy (Narosa)                                |

## Part - II (End of 2<sup>nd</sup> Year)

### PAPER -III

#### Group (A -55 Marks)

#### Section - I

#### (Analytical Geometry of two Dimensions)

1. (a) Transformations of rectangular axes: Translation, Rotation and their combinations. Theory of invariants. (2)

(b) General Equation of second degree in two variables: Reduction into canonical form. Classification of conics, Lengths and position of the axes. (2)

2. Pair of straight lines: Condition that the general equation of second degree in two variables may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by  $ax^2 + 2hxy + by^2 = 0$ . Angle bisector. Equation of two lines joining the origin to the points in which a line meets a conic. (3)

3. Circle, Parabola, Ellipse and Hyperbola: Equations of pair of tangents from an external point, chord of contact, poles and polars, conjugate points and conjugate lines. (4)

4. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole: Equations of tangent, normal, chord of contact. (5)

#### Section -II

#### (Analytical Geometry of three Dimensions) (30 Marks)

1. Rectangular cartesian co-ordinates in space. Halves and Octants. Concept of a geometric vector (directed line segment). Projection of a vector on a co-ordinate axis. Inclination of a vector with an axis. Co-ordinates of a vector. Direction cosines of a Vector. Distance between two points. Division of a directed line segment in a given ratio. (4)

2. Equation of a Plane: General form, Intercept and Normal form. The sides of a plane. Signed distance of a point from a plane. Equation of a plane passing through the intersection of two planes. Angle between two intersecting planes. Bisectors of angles between two intersecting planes. Parallelism and perpendicularity of two planes. (8)

3. Straight lines in space: Equation (Symmetric & Parametric form). Direction ratio and Direction cosines. Canonical equation of the line of intersection of two intersecting planes. Angle between two lines. Distance of a

point from a line. Condition of coplanarity of two lines. Equations of skew-lines. Shortest distance between two skew lines. (10)

4. Sphere (General Equation, Circle, Sphere through the intersection of two spheres, Radical Plane, Tangent, Normal)

Cone (Right circular cone, General homogeneous second-degree equation. Section of a cone by a plane as a curve and as a pair of lines, Condition for three perpendicular generators, Reciprocal cones, Developable cones, Frustums, cones parallel to axis of the axis, General form of equation, elliptic cone, cylinder, Enveloping cylinder).

Surface of Revolution (about axes of reference only). Ruled Surfaces. (15)

5. Transformation of rectangular axes by translation, rotation and their combinations. (2)

6. General equation of second degree in three variables. Reduction to canonical forms. Classification of Quadrics. (2)

7. Ellipsoid, Hyperboloid, Paraboloid: Canonical equations and the study of their shape. (5)

8. Tangent planes, Normals, Enveloping cone. (5)

9. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid. (8)

10. Knowledge of Cylindrical, Polar and Spherical polar co-ordinates their relations (No deduction required) (2)

#### BOOKS FOR REFERENCE

- |  |                  |
|--|------------------|
| 1. Co-ordinate Geometry                  | -S. L. Loney     |
| 2. Co-ordinate Geometry of Three         | -J. T. Brillouin |
| 3. Elementary Treatise on Conic Sections | -C. Smith        |
| 4. Solid Analytic Geometry               | -C. Smith        |

#### Group (B -45 Marks) (Analytical Dynamics of n Particles)

(Acquaintance with elementary concepts of Statics is assumed)

1. Fundamental Ideas and Principles of Dynamics. Laws of motion, Work, Power and Energy, Principles of conservation of energy and of momentums -Impulse and Impulsive forces. (5)

PAPER -IV

Group A -(25 Marks)

(Vector Algebra and Analysis)

1. Vector Algebra: Vector (directed line segment) Equality of two free vectors. Addition of vectors. Multiplication by a scalar.

Position vector, Point of division, Conditions of collinearity of three points and co-planarity of four points. Rectangular components of a vector in two and three dimensions.

Product of two or more vectors: Scalar and vector products. Scalar triple product and Vector triple products, Product of four vectors. Direct applications of Vector Algebra in (i) Geometrical and Trigonometrical problems, (ii) Work done by a force. Moment of a force about a point.

Vector equations of straight lines and planes. Volume of a tetrahedron. Shortest distance between two skew lines. (15)

2. Vector differentiation with respect to a scalar variable. Vector functions of one scalar variable. Derivative of a vector. Second derivative of a vector. Derivatives of sums and products. Velocity and Acceleration as derivative. (5)

3. Concepts of scalar and vector fields. Directional derivative. Gradient, Divergence and Curl, Laplacian. (5)

BOOKS FOR REFERENCE

- |                               |                                 |
|-------------------------------|---------------------------------|
| 1. Vector Analysis            | -Louis Brand                    |
| 2. Vector Analysis            | -Bart Spain                     |
| 3. Vector & Tensor Analysis   | -Spiegel (Schaum)               |
| 4. Elementary Vector Analysis | -C.E. Weatherburn (Vol. I & II) |

Group -B (40 Marks)

(Differential Equations)

1. Significance of ordinary differential equations: Geometrical and Physical consideration. Formation of differential equations by elimination of arbitrary constants. Meaning of the solution of ordinary differential equation.

Concept of linear and non-linear differential equations. (2)

2. Equations of first order and first degree: Statement of existence theorem. Separable, Homogeneous and Exact equations. Condition of exactness. Method of finding integrating factor. (5)

2. Motion in a straight line under variable acceleration, Motion under inverse square law, Composition of two S.H.M.'s of nearly equal frequencies. Motion of a particle tied to one end of an elastic string. Rectilinear motion in resisting medium, Damped forced oscillation. Motion under gravity where the resistance varies as some integral power of velocity, Terminal velocity. (10)

3. Impact of elastic bodies: Newton's experimental Law of elastic impact. Direct impact, Loss of K.E. in a direct impact. Oblique impact of two elastic spheres, Loss of K.E. in oblique impact. Angle of deflection. (3)

4. Expressions for velocity and acceleration of a particle moving on a plane in cartesian and polar co-ordinates. Motion of a particle moving in a plane with reference to a set of rotating axes. Motion of a particle in a plane. (6)

5. Central forces and central orbits. Characteristics of central orbits: Stability of nearly circular orbits. (4)

6. Tangential and Normal accelerations. Circular motion, Motion of a train or cyclist on a banked track, Simple cases of constrained motion of a particle. (4)

7. Motion of a particle in a plane under different laws of resistance, Motion of a projectile in a resisting medium in which the resistance varies as the velocity, Trajectories in a resisting medium where resistance varies as some integral power of the velocity. (5)

8. Motion on a smooth curve under resistance. (2)

9. Motion under inverse square law in a plane. Escape velocity, Planetary motion and Kepler's laws. Time of describing an arc of the orbit. Motion of artificial satellite. Slightly disturbed orbits. (6)

10. Conservative field of force and principle of conservation of energy. Motion of a rough curve (such as circle, parabola, ellipse, cycloid etc.) under gravity. (6)

11. Equation of motion of a particle of varying mass. Simple problems of motion of varying mass such as those of falling raindrops and projected rockets. (6)

#### BOOK FOR REFERENCE

1. An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies (S. L. Loney (Macmillan)

2. Motion in a straight line under variable acceleration, Motion under inverse square law, Composition of two S.H.M.'s of nearly equal frequencies. Motion of a particle tied to one end of an elastic string. Rectilinear motion in a resisting medium, Damped forced oscillation. Motion under gravity where its resistance varies as some integral power of velocity, Terminal velocity. (10)

3. Impact of elastic bodies; Newton's experimental law of elastic impact, Direct impact, Loss of K.E. in a direct impact, Oblique impact of two elastic spheres, Loss of K.E. in oblique impact, Angle of deflection. (3)

4. Expressions for velocity and acceleration of a particle moving on a plane in cartesian and polar co-ordinates, Motion of a particle moving in a plane with reference to a set of rotating axes, Motion of a particle in a plane. (6)

5. Central forces and central orbits, Characteristics of central orbits, Stability of nearly circular orbits. (4)

6. Tangential and Normal accelerations, Circular motion, Motion of a train or cyclist on a banked track, Simple cases of constrained motion of a particle. (4)

7. Motion of a particle in a plane under different laws of resistance, Motion of a projectile in a resisting medium in which the resistance varies as the velocity, Trajectories in a resisting medium where resistance varies as some integral power of the velocity. (5)

8. Motion on a smooth curve under resistance. (2)

9. Motion under inverse square law in a plane, Escape velocity, Planetary motion and Kepler's laws, Time of describing an arc of the orbit, Motion of artificial satellite, Slightly disturbed orbits. (6)

10. Conservative field of force and principle of conservation of energy, Motion of a rough curve (such as circle, parabola, ellipse, cycloid etc.) under gravity (6)

11. Equation of motion of a particle of varying mass, Simple problems of motion of varying mass such as those of falling raindrops and projected rockets. (6)

#### BOOK FOR REFERENCE

1. An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies - S.L. Loney (Macmillan)

3. First order linear equations: Integrating factor. Equations reducible to first order linear equations. (2)

4. Equations of first order but not of first degree: Clairaut's equation. Singular solution. (3)

5. Applications: Geometric applications, Orthogonal trajectories. (2)

6. Higher order linear equations with constant co-efficients: Complementary Function. Particular Integral. Method of undetermined co-efficients. Symbolic operator D. Method of variation of parameters.

Euler's homogeneous equation and Reduction to an equation of constant co-efficients. (8)

7. Second order linear equations with variable co-efficients :

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

Reduction of order when one solution of the homogeneous part is known. Complete solution. Method of variation of parameters. Reduction to Normal form. Change of independent variable. Operational Factors. (10)

8. Simple Eigen value problems. (2)

9. Simultaneous linear differential equations. Total differential equation: condition of integrability. (3)

10. Partial differential equation (PDE) : Introduction. Formation of P D E. Solution of PDE by Lagrange's Method of solution and by Charpit's Method. (5)

#### BOOKS FOR REFERENCE

- |  |   |
|--|---|
| 1. Differential Equations                                    | -Lester R Ford (Mc Graw Hill)                                     |
| 2. Differential Equations                                    | -Shepley L. Ross (John Wiley)                                     |
| 3. Differential Equations                                    | -H. T.H. Piaggio  |
| 4. A Text Book on Ordinary Differential Equations            | -Kisebejev, Makarenko, Krassov (Mir Publications)                 |
| 5. Differential Equations                                    | H.B. Phillips (John Wiley & Sons)                                 |
| 6. Differential Equations with applications and Programs     | S. Balachandra Rao, H. R. Anwartha (Universities Press)           |
| 7. Text Book of Ordinary Differential equations (2nd Edn)    | -S. G. Deo, V. Lakshminathan & V. Raghavendra (Tata Mc Graw Hill) |
| 8. An Elementary Course in Partial Differential Equations    | -T. Amamath (Newson)  |
| 9. An Introductory Course on Ordinary Differential Equations | D. A. Murray  |



3. First order linear equations: Integrating factor. Equations reducible to first order linear equations. (2)
4. Equations of first order but not of first degree: Clairaut's equation. Singular solution. (3)
5. Applications: Geometric applications, Orthogonal trajectories. (2)
6. Higher order linear equations with constant co-efficients: Complementary Function. Particular Integral. Method of undetermined co-efficients. Symbolic operator D. Method of variation of parameters. Euler's homogeneous equation and Reduction to an equation of constant co-efficients. (6)
7. Second order linear equations with variable co-efficients :
- $$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = F(x)$$
- Reduction of order when one solution of the homogeneous part is known. Complete solution. Method of variation of parameters. Reduction to Normal form. Change of independent variable. Operational Factors. (10)
8. Simple Eigen value problems. (2)
9. Simultaneous linear differential equations. Total differential equations: condition of integrability. (3)
10. Partial differential equation (PDE) : Introduction. Formation of P D E. Solution of PDE by Lagrange's Method of solution and by Charpit's Method. (5)

#### BOOKS FOR REFERENCE

- |   |   |
|---|---|
| 1. Differential Equations                                   | -Lester R Ford (Mc Graw Hill)                                       |
| 2. Differential Equations                                   | -Shepley L. Ross (John Wiley)                                       |
| 3. Differential Equations                                   | -H. T.H. Piaggio  |
| 4. A Text Book on Ordinary Differential Equations           | -Kiseleyev, Makarocko, Krzmov (Mir Publications)                    |
| 5. Differential Equations                                   | H.B. Phillips (John Wiley & Sons)                                   |
| 6. Differential Equations with applications and Programs    | S. Balachandra Rao, H. R. Anuradha (Universities Press)             |
| 7. Text Book of Ordinary Differential equations (2nd Ed.)   | -S. G. Deo, V. Lakshminarayanan & V. Raghavendra (Tata McGraw Hill) |
| 8. An Elementary Course in Partial Differential Equations   | -T. Amamath (Newson)  |
| 9. An Introductory Course on Ordinary Differential Equation | D. A. Murray  |

expectation, Covariance, Correlation co-efficient, Joint characteristic function, Multiplication rule for expectations. Conditional expectation. Regression curves, least square regression lines and parabolas. Chi-square and t-distributions and their important properties (Statements only) Tchebycheff's inequality. Convergence in probability. Statements of: Bernoulli's limit theorem, Law of large numbers, Poisson's approximation to binomial distribution and normal approximation to binomial distribution. Concepts of asymptotically normal distribution. Statement of central limit theorem in the case of equal components and of limit theorem for characteristic functions (Stress should be more on the distribution function theory than on combinatorial problems. Difficult combinatorial problems should be avoided). (40)

#### *Mathematical Statistics:*

Random sample. Concept of sampling and various types of sampling. Sample and population. Collection, tabulation and graphical representation. Grouping of data. Sample characteristic and their computation. Sampling distribution of a statistic. Estimates of a population characteristic or parameter. Unbiased and consistent estimates. Sample characteristics as estimates of the corresponding population characteristics. Sampling distributions of the sample mean and variance. Exact sampling distributions for the normal populations.

Divariate samples. Scatter diagram. Sample correlation co-efficient. Least square regression lines and parabolas. Estimation of parameters. Method of maximum likelihood. Applications to binomial, Poisson and normal population. Confidence intervals. Interval estimation for parameters of normal population. Statistical hypothesis. Simple and composite hypothesis. Best critical region of a test. Neyman-Pearson theorem (Statement only) and its application to normal population. Likelihood ratio testing and its application to normal population. Simple applications of hypothesis testing (for practical). (35)

*Theory of Errors:* (Statistical Table to be supplied).

#### **Group -B (50 Mark.)**

(Numerical Analysis and Computer Programming)

#### *Numerical Analysis:*

What is Numerical Analysis?

Errors in Numerical computation: Gross error, Round off error, Truncation error. Approximate numbers. Significant figures. Absolute, relative and percentage error.

Operators:  $\Delta, \nabla, E, \mu, \delta$  (Definitions and simple relations among them)

## Part – III (End of 3<sup>rd</sup> Year)

### PAPER-V

Group. A (60 Marks)

(REAL ANALYSIS, II)

1. Linear Point Set: Covering by open intervals, Sub-covering, Cantor intersection theorem, Lindelöf-covering theorem (statement only), Compact sets, Heine-Borel Theorem and its converse. (5)

2. Functions defined on point sets in one dimension: Limit and continuity, Continuity on compact set, Uniform continuity on compact set, Inverse function, Continuous image of compact set is compact. (5)

3.(a) Sequence of functions defined on a set  $(\subset \mathbb{R})$  : Pointwise and uniform convergence, Cauchy criterion of uniform convergence, Dini's theorem on uniform convergence, Weierstrass' M-test.

*Limit function:* Boundedness, Repeated limits, Continuity, Integrability and Differentiability of the limit function of a sequence of functions in case of uniform convergence.

(b) Series of functions defined on a set: Pointwise and uniform convergence, Cauchy criterion of uniform convergence, Dini's theorem on uniform convergence, Tests of uniform convergence -Weierstrass' M-test, Statement of Abel's and Dirichlet's test and their applications, Passage to the limit term by term, Sum function: boundedness, continuity, integrability, differentiability of a series of functions in case of uniform convergence.

(c) Power Series (P.S.) : Fundamental theorem of Power series, Cauchy-Hadamard theorem; Determination of radius of convergence, Uniform and absolute convergence of P.S. Properties of sum function, Abel's limit theorem, Uniqueness of power series having same sum function.

Exponential, logarithmic and trigonometric functions defined by Power Series and deduction of their salient properties. (20)

4. (a) Function of two variables,  $f: \mathbb{R}^2 \rightarrow \mathbb{R}^1$ , Mean value theorem and Taylor's theorem.

(b) Extremum of functions of two and three variables: Lagrange's Method of undetermined multipliers. (3)

5. Riemann Integration for bounded functions: Partition and refinement of partition of an interval, Upper Darboux sum  $U(P, f)$  & Lower Darboux sum  $L(P, f)$  and associated results, Upper Riemann (Darboux) integral and Lower Riemann (Darboux) integral, Darboux's theorem, Necessary and sufficient condition of R-integrability.

*Classes of Riemann Integrable functions:* Monotone functions, continuous functions, piecewise continuous functions with (i) finite number of points of discontinuities, (ii) infinite number of points of discontinuities having finite number of accumulation points.

expectation, Covariance, Correlation co-efficient, Joint characteristic function, Multiplication rule for expectations, Conditional expectation, Regression curves, least square regression lines and parabolas, Chi-square and t-distributions and their important properties (Statements only) Tchebycheff's inequality, Convergence in probability, Statements of: Bernoulli's limit theorem, Law of large numbers, Poisson's approximation to binomial distribution and normal approximation to binomial distribution, Concepts of asymptotically normal distribution, Statement of central limit theorem in the case of equal components and of limit theorem for characteristic functions (Stress should be more on the distribution function theory than on combinatorial problems. Difficult combinatorial problems should be avoided). (40)

*Mathematical Statistics:*

Random sample, Concept of sampling and various types of sampling, Sample and population, Collection, tabulation and graphical representation, Grouping of data, Sample characteristic and their computation, Sampling distribution of a statistic, Estimates of a population characteristic or parameter, Unbiased and consistent estimates, Sample characteristics as estimates of the corresponding population characteristics, Sampling distributions of the sample mean and variance, Exact sampling distributions for the normal populations.

Bivariate samples, Scatter diagram, Sample correlation co-efficient, Least square regression lines and parabolas, Estimation of parameters, Method of maximum likelihood, Applications to binomial, Poisson and normal population, Confidence intervals, Interval estimation for parameters of normal population, Statistical hypothesis, Simple and composite hypothesis, Best critical region of a test, Neyman-Pearson theorem (Statement only) and its application to normal population, Likelihood ratio testing and its application to normal population, Simple applications of hypothesis testing (for practical). (35)

*Theory of Errors:* (Statistical Table to be supplied).

**Group -B (50 Mark.)**

**(Numerical Analysis and Computer Programming)**

*Numerical Analysis:*

What is Numerical Analysis?

Errors in Numerical computation: Gross error, Round off error, Truncation error, Approximate numbers, Significant figures, Absolute, relative and percentage error.

Operators:  $\Delta, \nabla, E, \mu, \delta$  (Definitions and simple relations among them)

(ii) Determination of volume and surface area by Multiple Integrals  
(Problems only.) (5)

**Group -B (20 Marks)**  
**(METRIC SPACES)**

Definition and examples of Metric Spaces. Neighbourhoods. Limit points. Interior points. Open and closed sets. Closure and Interior. Boundary points. Sub-space of a Metric Space. Cauchy Sequences. Completeness. Cantor Intersection Theorem: Construction of real number as the completion of the incomplete metric spaces of rationals. Real number as a complete ordered field (No proof of theorem). Separable space:  $2^{\text{nd}}$  countable &  $\aleph_1$  Countable space. (10)

**Group -C (20 Marks)**  
**(COMPLEX ANALYSIS)**

Complex numbers as ordered pairs. Geometric representation of complex numbers. Stereographic projection.

Complex functions: Continuity and differentiability of complex functions. Analytic functions. Cauchy-Riemann Equations. Statement of Milne's Method, Harmonic functions. (10)

**PAPER -VI**  
**Group -A (50 Marks)**

**(Probability and Statistics)**  
*Mathematical Theory of Probability:*

Random experiments. Simple and compound events. Event space. Classical and frequency definitions of probability and their drawbacks. Axioms of Probability. Statistical regularity. Multiplication rule of probabilities. Bayes' theorem.

Independent events. Independent random experiments. Independent trials. Bernoulli trials and binomial law. Poisson trials. Random variables. Probability distribution. Distribution function. Discrete and continuous distributions. Binomial, Poisson, Gamma, Uniform and Normal distribution. Poisson Process (only definition). Transformation of random variables. Two dimensional probability distributions. Discrete and continuous distributions in two dimensions. Uniform distribution and two dimensional normal distribution. Conditional distributions. Transformation of random variables in two dimensions. Mathematical expectation. Mean, variance, moments, and central moments. Measures of location, dispersion, skewness and kurtosis. Median, mode, quantiles. Moment-generating function. Characteristic function. Two-dimensional

5. Stable and Unstable equilibrium. Co-ordinates of a body and of a system of bodies. Field of forces. Conservative field. Potential energy of a system. The energy test of stability. Condition of stability of equilibrium of a perfectly rough heavy body lying on fixed body. Rocking stones. (5)

6. Forces in three dimensions. Moment of a force about a line. Axis of a couple. Resultant of any two couples acting on a body. Resultant of any number of couples acting on a rigid body. Reduction of a system of forces acting on a rigid body. Resultant force is an invariant of the system but the resultant couple is not an invariant.

Conditions of equilibrium of a system of forces acting on a body. Deductions of the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. Poinsot's central axis. A given system of forces can have only one central axis. Wrench, Pitch, Intensity and Screw. Condition that a given system of forces may have a single resultant. Invariants of a given system of forces. Equation of the central axis of a given system of forces. (12)

### Group -C (20 Marks), (Rigid Dynamics)

Moment of ellipsoid. Equioscillatory system. Principal axis. D'Alembert's principle. D'Alembert's equations of motion. Principles of moments. Principles of conservation of linear and angular momentum. Independence of the motion of centre of inertia and the motion relative to the centre of inertia. Principle of energy. Principle of conservation of energy.

Equation of motion of a rigid body about a fixed axis. Expression for kinetic energy and moment of momentum of a rigid body moving about a fixed axis. Compound pendulum. Interchangeability of the points of a suspension and centre of oscillation. Minimum time of oscillation. Reaction of axis of rotation.

Equations of motion of a rigid body moving in two dimensions. Expression for kinetic energy and angular momentum about the origin of a rigid body moving in two dimensions. Two dimensional motion of a solid of revolution down a rough inclined plane. Necessary and sufficient condition for pure rolling. Two-dimensional motion of a solid of revolution moving on a rough horizontal plane.

Equations of motion under impulsive forces. Equation of motion about a fixed axis under impulsive forces. Centre of percussion. To show that (i) if there is a definite straight line such that the sum of the moments of the external impulses acting on a system of particles about it vanishes, then the total angular momentum of the system about that line remains unaltered, (ii) the change of K.E. of a system of particles moving in any manner under the application of impulsive forces is equal to the work done by the impulsive forces. Impulsive forces applied to a rigid body moving in two dimensions. (10)

12. Let us C : Yashvant Kaneekar (BPH Publications)
13. Programming in C : V. Krishnamoorthy and K. R. Radhakrishnan (Tata McGraw-Hill)
14. C by example: Noel Kalicharan (Cambridge University Press)
15. Programming in ANSI C: E. Balagurusamy (Tata McGraw-Hill)
16. Introduction to numerical analysis: F. B Hildebrand (TMH Edition)
17. Numerical Analysis: J. Scarborough
18. Introduction to numerical analysis: Carl Erik Froberg (Addison Wesley Publishing)
19. Numerical methods for science and engineering: R. G. Stanton (Prentice Hall)

**PAPER-VII**  
**Group -A (10 Marks)**  
**(Vector Analysis II)**

Line integrals as integrals of vectors, circulation, irrotational, vector, work done, conservative force, potential orientation. Statements and verification of Green's theorem, Stokes' theorem and Divergence theorem. (8)

**Group -B (35 Marks)**  
**(Analytical Statics)**

1. Friction: Laws of Friction, Angle of friction, Cone of friction. To find the positions of equilibrium of a particle lying on a (i) rough plane curve, (ii) rough surface under the action of any given forces. (4)

2. Centre of Gravity: General formula for the determination of C.G. Determination of position of C.G. of any etc, area of solid of known shape by method of integration. (3)

3. Astatic Equilibrium, Astatic Centre. Positions of equilibrium of a particle lying on a smooth plane curve under action of given forces. Action at a joint in a frame work. (4)

4. Virtual work: Principle of virtual work for a single particle. Deduction of the conditions of equilibrium of a particle under coplanar forces from the principle of virtual work. The principle of virtual work for a rigid body. Forces which do not appear in the equation of virtual work. Forces which appear in the equation of virtual work. The principle of virtual work for any system of coplanar forces acting on a rigid body. Converse of the principle of virtual work. (8)

3. Analytical Statics  
 4. Dynamics of a Particle and of Rigid bodies  
 5. Hydrostatics

-S. L. Loney  
 -S. L. Loney.  
 -A. S. Ramsay

**PAPER-VIII A**  
**Group -A (25 Marks)**  
**(Algebra II)**

*Section -1.. Linear Algebra (10 Marks)*

1. Linear Transformation (L.T.) on Vector Spaces: Definition of L.T., Null space, range space of an L.T., Rank and Nullity, Sylvester's Law of Nullity. [Rank (T) + Nullity (T) = dim (V)]. Determination of rank (T), Nullity (T) of Linear transformation  $T : R^n \rightarrow R^n$ . Inverse of Linear Transformation. Non-singular Linear Transformation.

Change of basis by Linear Transformation. Vector spaces of Linear Transformations. (5)

2. Linear Transformation and Matrices. Matrix of a linear Transformation relative to ordered bases of finite-dimensional vector spaces. Correspondence between Linear Transformations and Matrices. Linear Transformation is non-singular if its representative matrix be non-singular. Rank of L. T. = Rank of the corresponding matrix. (5)

*Section -2 : Modern Algebra (8 Marks).*

3. Normal sub-groups of a Group: Definition and examples. Intersection, union of Normal sub-groups. Product of a normal sub-group and a sub-group. Quotient Group of a Group by a Normal sub-group. (5)

4. Homomorphism and Isomorphism of Groups. Kernel of a Homomorphism. First Isomorphism Theorem. Properties deducible from definition of morphism. An infinite cyclic group is isomorphic to  $(\mathbb{Z}, +)$  and a finite cyclic group of order  $n$  is isomorphic to the group of residue classes modulo  $n$ . (5)

*Section -3 : Boolean Algebra (7 Marks)*

5. Boolean Algebra: Huntington Postulates for Boolean Algebra. Algebra of sets and Switching Algebra as examples of Boolean Algebra. Statement of principle of duality. Disjunctive normal and Conjunctive normal forms of Boolean Expressions. Design of single switching circuits. (10)



**Group -D (25 Marks)**  
**(Hydrostatics)**

1. Definition of fluid: Perfect fluid, Pressure. To prove that the pressure at a point in a fluid in equilibrium is the same in every direction. Transmissibility of liquid pressure. Pressure of heavy fluids. To prove

(i) In a fluid at rest under gravity the pressure is the same at all points in the same horizontal plane.

(ii) In a homogeneous fluid at rest under gravity the difference between the pressures at two points is proportional to the difference of their depths.

(iii) In a fluid at rest under gravity horizontal planes are surfaces of equal density.

(iv) When two fluids of different densities at rest under gravity do not mix, their surface of separation is a horizontal plane. Pressure in heavy homogeneous liquid. Theat of heavy homogeneous liquid of plane surfaces.

2. Definition of centre of pressure. Formulae for the depth of the centre of pressure of a plane area. Position of the centre of pressure. Centre of pressure of a triangular area whose angular points are at different depths. Centre of pressure of a circular area. Position of the centre of pressure referred to co-ordinate axes through the centroid of the area. Centre of pressure of an elliptical area when its major axis is vertical or along the line of greatest slope. Effect of additional depth on centre of pressure.

3. Equilibrium of fluids in given fields of force: Definition of field of force, line of force. Pressure derivative in terms of force. Surface of equal pressure. To find the necessary and sufficient conditions of equilibrium of a fluid under the action of a force whose components are  $X, Y, Z$  along the co-ordinate axes. To prove (i) that surfaces of equal pressure are the surfaces intersecting orthogonally the lines of force. (ii) when the force system is conservative, the surfaces of equal pressure are equi-potential surfaces and are also surfaces of equal density. To find the differential equations of the surfaces of equal pressure and density.

4. Rotating fluids: To determine the pressure at any point and the surfaces of equal pressure when a mass of homogeneous liquid contained in a vessel, rotates uniformly about a vertical axis.

5. Theory of gases: The atmosphere. Relations between pressure, density and temperature. Pressure in an isothermal atmosphere. Atmosphere in convective equilibrium. (10)

**BOOKS FOR REFERENCE**

L. Vessier Analysis  
L. Vessier Calculus

Spiegel (Mechanics)  
C. E. Weisbach

**Group -B (15 Marks)**  
**(Differential Equations II)**

1. Laplace Transform and its application in ordinary differential equations: Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Laplace Transform of derivatives. Laplace transform of integrals. Convolution theorem (Statement only). Application to the solution of ordinary differential equations of second order with constant coefficients.

2. Series Solution at an ordinary point: Power Series Solution of Ordinary differential equations, Simple Problems only.

**BOOKS FOR REFERENCE**

1. Advanced Calculus -David Widder (Prentice Hall)
2. Elementary Treatise on Laplace Transform -B. Sen (World Press)
3. Operational Methods in Applied Mathematics -H. S. Carslaw J .C. Jaeger.

**Group -C (20 Marks)**  
**(Tensor Calculus)**

A tensor as a generalized concept of a vector in an Euclidean space  $E^3$ . To generalize the idea in an n-dimensional space. Definition of  $E^n$ . Transformation of co-ordinates in  $E^n$  ( $n = 2, 3$  as example). Summation convention.

Contravariant and covariant vectors. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction. Outer and inner products of tensors. Quotient law. Reciprocal Tensor. Reciprocal space. Line element, and metric tensor. Reciprocal metric tensor. Raising and lowering of indices with the help of metric tensor. Associated tensor. Magnitude

of a vector. Inclination of two vectors. Orthogonal vectors. Christoffel symbols and their laws of transformations. Covariant differentiation of vectors and tensors.

(15)

**BOOKS FOR REFERENCE**

1. Tensor Calculus -Barry Spain
2. Vector Analysis and Tensor Calculus (Schaum Series) -Spiegel

PAPER -VIII B  
PRACTICAL (40 Marks)

*Numerical Analysis*

Newton's forward & backward interpolations, Stirling & Bessel interpolations, Lagrange's and Newton's Divided Difference Interpolations, Linear Interpolations.

Numerical differentiation based on Newton's forward and backward formulae.

Numerical integration: Trapezoidal and Weddle's rule.

Numerical solution of non-linear equations: Tabular, Bisection,

Secant/Regula Falsi and Fixed-point iterative methods.

Numerical solution of a system of linear equations: Gauss elimination method, Gauss Seidel iteration method, Matrix inversion by Gauss method.

*Statistics*

Sample characteristics -mean, variance, skewness, kurtosis, excess, mode, median, semi-interquartile range, Bivariate samples correlation coefficient regression lines, parabolic curve fitting, goodness of fit.

Confidence intervals for mean and standard deviation of a normal population. Approximate confidence limits for the parameter of a binomial population.

Tests of hypothesis -tests on mean and standard deviation of a normal population, comparison of means and standard deviations of two normal populations. Approximate tests on the parameter of a binomial population, on comparison of two binomial populations, Poisson distribution.

\* Above problems are to be done on a non-programmable scientific calculator.

The following problems are to be done on a computer using either FORTRAN C compiler

1. Numerical integration by Simpson's 3/8 rule.
2. Numerical solution of non-linear equations by Newton-Raphson method.
3. Numerical solution of ordinary differential equation by Runge-Kutta (4th order) method.

(Need not pass in individual group.)

**PAPER -VIII B**  
**PRACTICAL (40 Marks)**

*Numerical Analysis*

Newton's forward & backward interpolations, Stirling & Bessel interpolation, Lagrange's and Newton's Divided Difference Interpolation, Inverse Interpolation.

Numerical differentiation based on Newton's forward and backward formulae.

Numerical integration: Trapezoidal and Weddle's rule.

Numerical solution of non-linear equations: Bisection, Secant/Regula Falsi and Fixed-point iteration methods.

Numerical solution of a system of linear equations: Gauss elimination method, Gauss Seidel iteration method, Matrix inversion by Gauss method.

*Statistics*

Sample characteristics -mean, variance, skewness, kurtosis, excess, mode, median, semi-interquartile range. Bivariate samples -correlation coefficient, regression lines, parabolic curve fitting, goodness of fit.

Confidence intervals for mean and standard deviation of a normal population. Approximate confidence limits for the parameter of a binomial population.

Tests of hypothesis -tests on mean and standard deviation of a normal population, comparison of means and standard deviations of two normal populations. Approximate tests on the parameter of a binomial population, on comparison of two binomial populations. Poisson distribution.

\* *Above problems are to be done on a non-programmable scientific calculator.*

The following problems are to be done on computers using either FORTRAN C compiler

1. Numerical integration by Simpson's 1/3 rule.
2. Numerical solution of non-linear equation by Newton-Raphson method.
3. Numerical solution of ordinary differential equation by Runge-Kutta (4th order) method.

(Need not pass in individual group.)

### Three Dimensions :

1. Rectangular Cartesian co-ordinates: Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.
2. Equation of a Plane: General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.
3. Equations of Straight line: General and symmetric form. Distance of a point from a line. Coplanarity of two straight lines. Shortest distance between two skew-lines.
4. Sphere and its tangent plane.
5. Right circular cone.

### Group B: 15 Marks (Vector Algebra)

Addition of Vectors. Multiplication of a Vector by a scalar. Collinear and Coplanar Vectors. Scalar and Vector products of two and three vectors. Simple applications to problems of Geometry. Vector equation of plane and straight line. Volume of Tetrahedron. Applications to problems of Mechanics (Work done and Moment)

### PAPER-III

100 Marks

### Group A: 50 Marks (Differential Calculus)

1. Rational numbers. Geometrical representations. Irrational number. Real number represented as point on a line -Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included)
2. Sequence: Definition of bounds of a sequence and monotonic sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotonic sequences -applications of the theorems, in particular definition of  $\epsilon$ . Statement of Cauchy's general principle of convergence and its

**Group II: 25 Marks**  
**(Modern Algebra)**

1. Basic concept: Sets, Sub-sets, Equality of sets, Operations on sets: Union, Intersection and Complement. Verification of the laws of Algebra of sets and De Morgan's Laws. Cartesian product of two sets.  
Mappings, One-One and onto mappings. Composition of Mappings - concept only. Identity and Inverse mappings. Binary Operations in a set, Identity element, Inverse element.

2. Introduction of Group Theory. Definition and examples taken from various branches (examples from number systems, roots of unity,  $2 \times 2$  real matrices, non-singular real matrices of a fixed order). Elementary properties using definition of Group. Definition and examples of sub-group. Statement of necessary and sufficient condition for applications.

3. Definitions and examples of (i) Ring, (ii) Field, (iii) Sub-ring, (iv) Sub-field.

4. Concept of Vector space over a Field: Examples. Concepts of Linear combinations, Linear dependence and independence of a finite set of vectors, Sub-space. Concepts of generators and basis of a finite-dimensional vector space. Problems on formation of basis of a vector space (No proof required)

5. Real Quaternions: Form involving not more than three variables - Problems only.

**Paper II**

**Group A: 35 Marks**  
**(Analytical Geometry of two and three dimensions)**

**Two Dimensions:**

1. Transformations of Rectangular axes: Translation, Rotation and Shear operations. Invariance.

2. General equation of second degree in  $x$  and  $y$ : Reduction to canonical form. Classification of conic.

3. Pair of straight lines: Condition that the general equation of 2nd degree in  $x$  and  $y$  may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by  $\tan^{-1} \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2} = 0$ . Equation of bisectors. Equation of two lines intersecting the origin by the points in which a line meets  $x$ -axis.

4. Equation of pair of tangents drawn from external point, chord of contact, poles and polars in case of Central conic: Particular cases for Parabola, Ellipse, Circle, Hyperbola.

5. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

## Part II (End of Second Year)

### Paper IV

Group A: 30 Marks  
(Integral Calculus)

1. Integration of the form: -

$\int \frac{dx}{a + b \cos x}$ ,  $\int \frac{l \sin x + m \cos x}{n \sin x + p \cos x} dx$  and Integration of Rational functions.

2. Evaluation of definite Integrals.

3. Integration as the limit of a sum (with equally spaced as well as unequal intervals).

4. Reduction formulae of  $\int \sin^m x \cos^n x dx$ ,  $\int \frac{\sin^m x}{\cos^n x} dx$ ,  $\int \tan^n x dx$  and associated problems (m and n are non-negative integers)

5. Definition of Improper Integrals: Statements of (i)  $p$ -test, (ii) Comparison test (Limit form excluded) - Simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).

6. Working knowledge of Double integral.

7. Applications: Rectification, Quadrature, Volume and Surface areas of solids formed by revolution of plane curve and areas - Problems only.

Group B: 20 Marks  
(Ordinary Differential Equations)

1. Order, degree and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE:

First order equations:

(i) Variables separable

(ii) Homogeneous equations and equations reducible to homogeneous forms.

(iii) Exact equations and those reducible to such equation.

(iv) Euler's and Bernoulli's equations (Linear).

(v) Clairaut's Equations: General and Singular solutions.

2. Second order linear equations:

Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

3. Simple applications: Orthogonal Trajectories.

3. Infinite series of constant terms: Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms: Statements of Comparison test, D. Alembert's Ratio test, Cauchy's  $n$ th root test and Raabe's test -Applications. Alternating series: Statement of Leibnitz test and its applications.

4. Real-valued functions defined on an interval; Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval.

Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.

5. Derivative -its geometrical and physical interpretation. Sign of derivative - Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential -application in finding approximation.

6. Successive derivative -Leibnitz's Theorem and its application.

7. Statement & Proof Rolle's Theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like

$e^x$ ,  $\sin x$ ,  $\cos x$ ,  $(1+x)^n$ ,  $\log(1+x)$  [with restrictions wherever necessary]

8. Indeterminate Forms: L'Hospital's Rule: Statement and problems only.

9. Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.

10. Functions of two and three variables: Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives: Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Functions of two variables -Successive partial derivatives: Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables.

-Maxima and minima of functions of not more than three variables -Lagrange's Method of undetermined multiplier -Problems only. Implicit function in case of function of two variables (existence assumed) and derivative.

11. Applications of Differential Calculus: Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only). Curvature of plane curves. Envelope of family of straight lines and of curves (Problems only). Definitions and examples of singular points (viz. Node, Cusp, Isolated point).



3. Infinite series of constant terms: Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms: Statements of Comparison test, D. Alembert's Ratio test, Cauchy's  $n$ th root test and Raabe's test -Applications. Alternating series: Statement of Leibnitz test and its applications.
4. Real-valued functions defined on an interval: Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval.  
Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.
5. Derivative -its geometrical and physical interpretation. Sign of derivative - Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential -application in finding approximation.
6. Successive derivative -Leibnitz's Theorem and its application.
7. Statement & Proof Rolle's Theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $(1+x)^n$ ,  $\log(1+x)$  (with restrictions wherever necessary)
8. Indeterminate Forms: L'Hospital's Rule: Statement and problems only.
9. Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.
10. Functions of two and three variables: Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives: Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Functions of two variables -Successive partial derivatives: Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables.  
-axima and minima of functions of not more than three variables -Lagrange's Method of undetermined multiplier -Problems only. Implicit function in case of function of two variables (existence assumed) and derivative.
11. Applications of Differential Calculus: Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only). Curvature of plane curves. Envelope of family of straight lines and of curves (Problems only). Definitions and examples of singular points (viz. Node, Cusp, Isolated point).

## Part – III (End of 3<sup>rd</sup> Year)

PAPER -VII

100 Marks

*(ANY TWO OF THE FOLLOWING GROUPS)*

### Group A : 50 marks (Elements of Computer Science and Programming)

**A. Boolean algebra** -Basic Postulates and Definition. Two-element Boolean algebra. Boolean Function. Truth table. Standard forms of Boolean function - DNF and CNF. Minterms and maxterms. Principle of Duality. Some laws and theorem of Boolean algebra. Simplification of Boolean expressions - Algebraic method and Karnaugh Map method. Application of Boolean algebra -Switching Circuits, Circuit having some specified properties, Logical Gates -AND, NOT, OR, NAND, NOR, etc.

**B. Computer Science and Programming:** Historical Development, Computer Generation, Computer Anatomy -Different Components of a Computer System, Operating System, Hardware and Software.

Positional Number System: Binary to Decimal and Decimal to Binary. Other systems. Binary Arithmetic. Octal, Hexadecimal, etc. Storing of data in a Computer -BIT, BYTE, WORD, etc. Coding of a data -ASCII, etc.

Programming Language: Machine language, Assembly language and High level language. Compiler and Interpreter. Object Programme and Source Programme. Ideas about some H.L. -e.g. FORTRAN, C, 90 or

COBOL, PASCAL, etc.

Algorithms and Flow Charts -their utilities and important features. Ideas about the complexities of an algorithm. Application in simple problems. FORTRAN 77/90 : Introduction, Data Type -Keywords, Constants and variables -Integer, Real, Complex, Logical, Character, Subscripted Variables, Fortran Expressions.

I/O statements -formatted and unformatted Programme execution control Logical if, if-then-else, etc. Arrays, dimension statement Repetitive Computation -Do, Nested Do etc.

Sub Programs -(i) Function Sub Programme

(ii) Subroutine Sub Programme

Elements of BASIC Programming Language: Reading, Printing, Branch & Loop, Array, Functions.

Application to Simple Problems. An exposure to M.S. Office, e-mail, Internet (Through Demonstration only).

**Group B (50 Marks)**  
**(Probability & Statistic)**

*(Emphasis on Applications only)*

A. Elements of Probability Theory: Random experiment, Outcome, Event, Mutually Exclusive Events, Equally likely and Exhaustive. Classical definition of Probability, Theorems of Total Probability, Conditional Probability and Statistical Independence. Bayes' Theorem. Problems. Shortcomings of the classical definition. Axiomatic approach-Problems, Random Variable and its Expectation. Theorems on mathematical expectation. Joint distribution of two random variables. Theoretical Probability Distribution -Discrete and Continuous (p.m.f. p.d.f.) Binomial, Poisson and Normal distributions and their properties.

B. Elements of Statistical Methods. Variables, Attributes. Primary data and secondary data. Population and sample. Census and Sample Survey. Tabulation - Chart and Diagram, Graph, Bar diagram, Pie diagram etc. Frequency Distribution -Un-grouped and grouped cumulative frequency distribution. Histogram, Frequency curve, Measures of Central Tendencies

-Averages: AM, OM, HM, Mean, Median and Mode (their advantages and disadvantages) Measures of Dispersions -Range, Quartile Deviation, Mean Deviation, Variance/S.D., Moments, Skewness and Kurtosis.

C. Sampling Theory: Meaning and objects of sampling. Some ideas about the methods of selecting samples. Statistic and Parameter, Sampling Distribution - standard error of a statistic (e.g. sample mean, sample proportion). Four fundamental distributions derived from the normal: (i) Standard Normal Distribution, (ii) Chi-square distribution (iii) Student's distribution (iv) Snedecor's F-distribution.

D. Bivariate Frequency Distribution. Scatter Diagrams, Correlation co-efficient - Definition and properties. Regression lines.

**Group- C Elements of Difference Equation And calculus of Variation (50, Marks).**

## BOOKS FOR REFERENCE

1. Fortran 77 and Numerical Methods: C. Xavier;  
(Wiley Eastern Limited/New Age International Limited)
2. Programming and Computing with Fortran 77/90: P. S. Grover;  
(Allied Publishers Limited)
3. Structured FORTRAN 77 for Engineers and Scientists: D. M. Etter  
(The Benjamin/Cummings Publishing Co. Inc)
4. Programming in Basic : Balagurusamy.

### Group B(50 marks) (A Course of Calculus)

1. Determination of Radius of convergence of Power Series.

Statement of properties of continuity of sum function of power series, Term by term integration and Term by term differentiation of Power Series. Statements of Abel's Theorems on Power Series. Convergence of Power Series. Expansions of elementary functions such as  $e^x$ ,  $\sin x$ ,  $\log(1+x)$ ,  $(1+x)^n$ . Simple problems.

2. Fourier series on  $(-\pi, \pi)$  : Periodic function, Determination of Fourier coefficients. Statement of Dirichlet's conditions of convergence and statement of the theorem on convergence of Fourier Sine and Cosine series.

3. Third and Fourth order ordinary differential equation with constant coefficients. Euler's Homogeneous Equation.

4. Second order differential equation: (a) Method of variation of parameters. (b) Method of undetermined co-efficients. (c) Simple eigenvalue problem.

5. Simultaneous linear differential equation with constant co-efficients.

6. Laplace Transform and its application to Ordinary differential equation. Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Application to the solution of ordinary differential equation of second order with constant coefficients.

7. Partial Differential Equation (PDE) : Introduction, Formation of PDE, Solutions of PDE, Lagrange's method of solution.