U.G. 5th Semester Examination 2021

MATHEMATICS (General)

Paper : DSE-1

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

DSE-1A

[Abstract & Linear Algebra]

Group - A

(4 Marks)

1. Answer any ***four*** questions : $4\times1=4$

   (a) Show that the set of vectors $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ is linearly independent in $\mathbb{R}^3$.

   (b) State Cayley-Hamilton theorem for a square matrix.

   (c) Find the eigenvalues of the matrix : $A=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$.

   (d) What is the order of the symmetric group S_n?

   (e) Let $T:\mathbb{R}\to\mathbb{R}$ defined by $T(x)=x^2$. Is T a linear transformation?

   (f) Find all the unit elements in the ring $(\mathbb{Z}_8, +, ...)$.

   (g) Find the power set of the set $S=\{1, 2, 3\}$.

Group - B

(10 Marks)

Answer any *two* questions : 2×5=10

2. (a) Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 5x + 1$. Examine if f is a bijective function.

   (b) A relation ρ is defined on the set $\mathbb{Z}$ by "$a\rho b$ if and only if $a - b$ is divisible by 5" for $a, b \in \mathbb{Z}$. Examine if ρ is an equivalance relation on $\mathbb{Z}$. 2+3

3. Prove that the set $\mathbb{Z}_5 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ forms a group with respect to addition modulo 5.

   5

4. Show that $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y, y + z, z + x), (x, y, z) \in \mathbb{R}^3$ is a linear transformation. Find $KerT$ and ImT. 5

5. Find the eigenvalues and the corresponding eigenvectors of the matrix :

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$$ 5

Group - C

(18 Marks)

Answer any *two* questions : 2×9=18

6. (a) Prove that the characteristic of an integral domain is either zero or a prime number. 5

   (b) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by

   $$T(x, y, z) = (x + y, y + z), (x, y, z) \in \mathbb{R}^3$$

   Find the matrix of T relative to the ordered bases {(1, 0, 0), (0, 1, 0), (0, 0, 1)} of $\mathbb{R}^3$ to the ordered bases {(1, 0), (0, 1)} of $\mathbb{R}^2$. 4

7. (a) Use Cayley-Hamilton theorem to find A^{50}, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. 5

(b) Prove that the set $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b+c+d=0 \right\}$ is a subspace of $M_2(\mathbb{R})$. 4

8. (a) Prove that the function $< , > : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$< \boldsymbol{u}, \boldsymbol{v} > = 2u_1v_1 + u_1v_2 + u_2v_1 + u_2v_2$ for all $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^2$.

where $\boldsymbol{u} = (u_1, u_2)$ and $\boldsymbol{v} = (v_1, v_2)$ is an inner product on $\mathbb{R}^2$. 4

(b) Prove that a field is an integral domain. Give an example of an integral domain which is not a field. 4+1

DSE-1B
[Differential Equation II & Mechanics]

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
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Group - A

(4 Marks)

1. Answer any ***four*** questions : $4\times1=4$

(a) Determine whether $x=0$ is an ordinary point or regular singular point of the differential equation $2x^2D^2y+7x(x+1)Dy+3y=0$, $D\equiv\frac{d}{dx}$.

(b) Solve $xp-yq=xy$ by Lagrange's method.

(c) Obtain partial differential equation from $z=f(\sin x+\cos y)$.

(d) The velocity of a particle moving in a straight line at any time instant 't' when its distance from the origin x, is given by $x=\frac{1}{2}v^2$. Show that the acceleration of the particle is constant.

(e) Find a complete integral of $z=pq$.

(f) What is degrees of freedom.

(g) What is the order of the equation $xy^3\left(\frac{\partial y}{\partial x}\right)^2+yx^2+\frac{\partial y}{\partial x}=0$?

Group - B

(10 Marks)

Answer any *two* questions : 2×5=10

2. Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ in powers of x. (about a point $x=0$)

3. Use charpits method, find a complete integral of $pxy+pq+qy-yz=0$.

4. A particle is projected vertically upwards with a velocity v from earths surface. If h and H be the greatest heights attained by the particle moving under uniform and variable accelerations respectively. Then show that $\frac{1}{h}-\frac{1}{H}=\frac{1}{R}$, where R = radius of earth.

5. Let t_1 and t_2 be the periods of vertical oscillations of two different weights suspended by an elastic string and c_1 and c_2 be the statical extensions due to these weights. Prove that $g(t_1^2-t_2^2)=4\pi(c_1-c_2)$.

Group - C

(18 Marks)

Answer any *two* questions : 2×9=18

6. (a) A particle moves under a force $m\mu\{3au^4-2(a^2+b^2)u^5\}$ $(a>b)$ and is projected from an apne at a distance $a+b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit $r=a+b\cos\theta$.

   (b) The middle points of opposite sides of quadrilatteral formed by four freely jointed weightless bars are connected by two light rods of lengths l and l' in a state of tension. If T, T' be the tension in these rods prove that $\frac{T}{l}+\frac{T'}{l'}=0$. 5+4

7. (a) Find the complete solution of the equation $z = px + qy + p^2 + q^2$ by Charpit's method. 5

(b) Find the eigen value and corresponding eigen functions of $\frac{d^2y}{dx^2} + \lambda y \; dx = 0 \, (\lambda > 0)$ with the boundary conditions $y(0) = 1, \; y'(\pi) = 0$. 4

8. (a) If the radial and cross radial vlocities of a particle be respectively $\mu\theta$ and λr. Show that the path of a particle can be represented by an equation of the form $r = A\theta^2 + B$. 5

(b) A particle describe a parabola $r = a\sec^2\left(\frac{\theta}{2}\right)$ such that cross-radial velocity is constant. Show that $\frac{d^2r}{dt^2}$ is constant. 4