UG 1st Semester Examination 2021

MATHEMATICS (Honours)

Paper : DC-2

[Algebra]

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) State the De Moivre's theorem.

(b) Prove that $e^n > \frac{(n+1)^n}{n!}$, where $n \in \mathbb{N}$.

(c) Show that a polynomial of odd degree with real coefficients must have at least one real root.

(d) Let X and Y be two finite sets having m and n elements respectively. What will be the number of distinct relations that can be defined from X to Y?

(e) If $a|c$ and $b|c$ with $gcd(a, b) = 1$, show that $ab|c$.

(f) Find the value of a and b so that the four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0), (0, 1, a, b) are linearly independent.

(g) The matrix $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ has two eigen values 1 and 5. Find the eigen vector corresponding to the eigen value 5.

Group - B

(10 Marks)

Answer any *two* questions : 2×5=10

2. Express $\frac{-1+i\sqrt{3}}{1+i}$ in polar form and then deduce the value of $\cos\frac{5}{12}\pi$. 5

3. If $a_1, a_2, \ldots\ldots, a_n$ be n positive numbers and $a_n a_{n-1} = 1$, then show that

$$\left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^n \geq \left(\frac{a_1 + a_2 + a_3 + \ldots + a_{n-2}}{n-2}\right)^{n-2}.$$ 5

4. Solve $x^3 - 6x - 9 = 0$ by Cardan's method. 5

5. Let $S = \{x \in R : -1 < x < 1\}$ and $f : R \to S$ be defined by $f(x) = \frac{x}{1+|x|}, x \in R$. Show that f is a bijection. Determine f^{-1}. 5

Group - C

(18 Marks)

Answer any *two* questions : 2×9=18

6. (a) Let $f : A \to B$ be a bijective mapping. Then show that the mapping $f^{-1} : B \to A$ is also a bijection and $\left(f^{-1}\right)^{-1} = f$. 4

   (b) Satate the Fundamental theorem of arithmetic. Find the G.C.D of 792 and 385 and express it in the form $792l + 385m$. 2+3

7. (a) Prove that the eigen vectors corresponding to two distinct eigen values of a real symmetric matrix are orthogonal. 4

   (b) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then verify that A satisfies its own characteristic equation. Hence find A^9. Find also A^{-1}. 5

8. (a) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

$$\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right).$$ 4

(b) If $x = \cos\theta + i\sin\theta, y = \cos\phi + i\sin\phi$, then prove that $\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$, where m and n are integers. 5
