

GOUR MAHAVIDYALAYA 1ST UNIT TEST

MATHEMATICS (Honours)

Paper Code: DC-H-13

Full Marks: 20

Time: One Hour

Group-A
(Marks 20)

Answer any four questions.

$4 \times 5 = 20$

1. Define Convex set, Convex hull and Convex polyhedron in E^n . If x_1, x_2 be real number.

Show that the set $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. [5]

2. Define Extreme point of a set and .Find the extreme point, if any of the following sets. [5]

(a) $S = \{(x, y) : x^2 + y^2 \leq 16\}$

(b) $P = \{(x, y) : |x| \leq 1, |y| \leq 1\}$.

(c) $W = \{(x, y) : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$.

3. State fundamental theorem of L.P.P.

If $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the L.P.P.

$$\text{Maximize } z = x_1 + 2x_2 + 4x_3$$

Subject to

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$

find a basis feasible solution. [5]

4. Find the optimal solution to the following transportation problem: [5]

	W_1	W_2	W_3	W_4	
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
	5	8	7	14	

5. Find the optimal assignment to find the minimum cost for the assignment problem with the following matrix [5]

	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

2022
Gour Mahavidyalaya
MATHEMATICS (Honours)
Paper Code: MATH-H-DSE 03
Point set topology
[CBCS]

Full Marks : 20

Time : 1 hr 15 minutes

The figures in the margin indicate full marks.
 Notations and symbols have their usual meanings.

Answer any two questions from 1 to 4.

5×2=10

1. Define subbasis for a topology on X . Give example.

Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A .

[5]

2. When two topologies on X are said to be comparable?

The topologies of \mathbb{R}_l (lower limit topology) and \mathbb{R}_k (K-topology) are not comparable.

[5]

3. If $\{\tau_\alpha : \alpha \in I\}$ be a family of topologies on X , show that $\bigcap_{\alpha \in I} \tau_\alpha$ is a topology on X

Is $\bigcup_{\alpha \in I} \tau_\alpha$ a topology on X ? Justify.

[5]

4. Consider the set $Y = [-1, 1]$ as a subspace of \mathbb{R} . Which of the following sets are open in Y ?
 Which are open in \mathbb{R} ?

$$A = \{x : \frac{1}{2} < |x| < 1\}$$

$$B = \{x : \frac{1}{2} < |x| \leq 1\}$$

$$C = \{x : \frac{1}{2} \leq |x| < 1\}$$

$$D = \{x : \frac{1}{2} \leq |x| \leq 1\}$$

Discuss.

[5]

5. Answer any two questions.

4×2=8

(a) Prove that only connected subsets of \mathbb{Q} are singletons in the subspace topology of \mathbb{R} .

[4]

(b) Prove that connectedness is a topological property.

[4]

(c) Let X and Y be two connected topological spaces. Show that the product space $X \times Y$ is also connected.

[4]

6. Answer any one question.

2×1=2

(a) Show that \mathbb{R} is connected in cofinite topology.

[2]

(b) Let τ and τ' are topological spaces on X . If $\tau' \supset \tau$ What does connectedness of X in topology imply about connectedness in the other?

[2]