## Gour Mahavidyalaya

**MATHEMATICS** (Honours)

Paper Code: MATH-H-DC03

[CBCS]

Full Marks: 20 Time: 1:30 hours The figures in the margin indicate full marks. Notations and symbols have their usual meanings. Group - A 1. Is the set  $A = \{x \in \mathbb{R} : 0 \le x \le 1\}$  enumerable? Justify your answer. [2] 2. Show that, a non-empty subset of an enumerable set is countable. [2] 3. Let, *f* be a function defined on  $\mathbb{R}$  by,  $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0. \end{cases}$ Show that f is differentiable at 0 but f' is not continuous at 0. [3] 4. Define perfect set. Show by an example that continuity of a function does not ensure it's differentiability. [3] Group - B (Answer any five) 1. Test the convergence of the sequence  $(x_n)$ , where  $x_n = (\sqrt{2} - 2^{\frac{1}{3}}) (\sqrt{2} - 2^{\frac{1}{5}}) \dots (\sqrt{2} - 2^{\frac{1}{2n+1}})$ [Hint: Sandwich theorem, you can use the fact  $2^{\frac{1}{2n+1}} \ge 1 \ \forall n \in \mathbb{N}$ ] [2] 2. State the Bolzano-Weierstrass theorem for sequence. Give an example of an unbounded sequence that has a convergent subsequence. [2]

- 3. Let a > 0 and  $x_1 > 0$ . Define  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$  for all  $n \in \mathbb{N}$ . Show that the sequence  $(x_n)$  is convergent and converges to  $\sqrt{a}$ . [2]
- 4. Show that

$$\log\left(\frac{5}{3}\right) = \frac{1}{2} \left[ 1 + \frac{1}{3} \left(\frac{1}{4}\right)^2 + \frac{1}{5} \left(\frac{1}{4}\right)^4 + \cdots \right]$$

[Hint:  $\log \frac{1+x}{1-x} = \log(1+x) - \log(1-x)$ ]

5. Test the convergence of

$$\sum_{n=1}^{\infty} \left[ \frac{1}{n} - \log\left(\frac{n+1}{n}\right) \right]$$

[Hint: take  $b_n = \frac{1}{n^2}$ , comparison test]

- 6. If  $\sum \frac{1}{n^2} = S$ , Prove that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{3}{4}S$ [Hint: since,  $\sum \frac{1}{n^2}$  is absolute convergent, the sum of the series can't be altered.] [2]
- 7. Show that, every Cauchy sequence is bounded. Is the converse true? Justify.
- 8. let  $f_n$  be the fibonacci sequence and let  $x_n = \frac{f_{n+1}}{f_n}$ . Suppose that  $l = \lim x_n$ . What is the value of *l*? [Hint: use definition of fibonacci sequence.] [2]

[2]

[2]