GOUR MAHAVIDYALAYA

MATHEMATICS (Honours)

Paper Code: MATH-DC01

Semester-I

Class Test-I

Full Marks : 20 Time : 60 minutes

1. Answer all the question. [5]

(a) State the necessary and sufficient condition for the general equation of second degree to represents a pair of real straight lines. [1]

(b) Show that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ represents a pair of parallel lines and find the distance between them. [2]

(c) Use Cauchy's principle to show that $\lim\limits_{x\to 0} \cos\frac{1}{x}$ does not exists. [1]

(d) Evaluate $\lim\limits_{x\to 0^+} \{\frac{\sin x}{x}\}$, $\lim\limits_{x\to 0^-} \{\frac{\sin x}{x}\}$, where $\{x\} = x - [x] =$ fractional part of x for all $x \in \mathbb{R}$. [1/2+1/2]

2. Answer any three questions. 5× 3=15

(a) Show that if one of the lines given by the eqation $ax^2 + 2hxy + by^2 = 0$ be perpendicular to one of the lines given by $a'x^2 + 2h'xy + b'y^2 = 0$ then $(aa' - bb')^2 + 4(ah' + hb')(ha' + bh') = 0$. [5]

(b) i. If the pair of straight lines joining the origin to the points of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $lx + my + n = 0$ are perpendicular to each other, then show that $\frac{a^2+b^2}{l^2+m^2} = \frac{a^2b^2}{n^2}$.

ii. The axes are rotated through an angle of 60° without change of origin. The coordinates of a point are $(4, \sqrt{3})$ in the new coordinate system. Find the coordinate of if in the old system.

[3+2]

(c) A function $f : [0, 1] \to \mathbb{R}$ is defined by $f(x) = x,\ x$ is rational in $[0, 1]$ $f(x) = (1 - x),\ x$ is rational in $[0, 1]$. show that (i) f is injective on $[0, 1]$, (ii) f assumes every real number in $[0, 1]$ (iii) f is continuous at $1/2$ and discontinuous at every other point in $[0, 1]$ [1+2+2]

(d) Define continuity of a function on a domain D.
A function $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and $f(\frac{x+y}{2}) = \frac{f(x)+f(y)}{2}$ for all $x, y \in \mathbb{R}$. Prove that $f(x) = ax + b, (a, b \in \mathbb{R})$. [2+3]

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Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: MATH-H-DC02

Full Marks : 20 Time : 1 hour

1. (a) Define a partition of a set with an example. Let $f : \mathbb{R} \to \mathbb{R}$ be a mapping defined by $f(x) = x^2, x \in \mathbb{R}$. Is it surjective? [2]

   (b) Give an example of a relation on a set
   (i) Which is symmetric, transitive but not reflexive.
   (ii) Which is symmetric but neither transitive nor reflexive. [1+1]

2. Any two. $2 \times 3 = 6$

   (a) Let S be set containing n elements. How many different reflexive relation can be defined on S? [3]

   (b) Let P be an equivalence relation on a set A and a,b $\in$ A, a $\bar{\rho}$ b. Show that $[a] \cap [b] = \phi$ [3]

   (c) Let $f : \mathbb{R} \to S$ be a mapping defined by $f(x) = \frac{x}{1+|x|}, x \in \mathbb{R}$ where $S = \{x \in \mathbb{R} : -1 < x < 1\}$. Show that f is bijective. [3]

3. Answer any one.

   (a) i. If a, b, c are three real numbers then show that $a^4 + b^4 + c^4 \geq abc(a + b + c)$ When equal sign occurs?
   ii. Prove that, $\sqrt{n} < (n!)^{\frac{1}{n}} < \frac{n+1}{2}$ for $n \geq 3, n \in \mathbb{N}$ [2+3]

   (b) i. If a, b, c are lengths of the sides of a triangle then show that $(1 + \frac{b}{a} - \frac{c}{a})^a(1 + \frac{c}{b} - \frac{a}{b})^b(1 + \frac{a}{c} - \frac{b}{c})^c < 1$
   ii. If $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$ then show that $abc < \frac{1}{8}$ [2+3]

4. Answer any one.

   (a) i. Find the value of $\sum_{n=1}^{13}(i^n + i^{n-1})$, where $i = \sqrt{-1}$
   ii. Find the product of all values of $(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^{\frac{3}{4}}$ [2+3]

   (b) Write De Moivre's theorem.
   Find the principal argument of $1 - \sin\alpha + i\cos\alpha, \alpha \in [0, 2\pi)$ [1+4]

TUTORIAL/UG/1st Sem/H/22/GM (CBCS)

Gour Mahavidyalaya

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