U.G. 6th Semester Examination 2022

PHYSICS (Honours)

Paper Code : DSE 3 - A & B

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

DSE 3 - A

(Advanced Mathematical Methods - II)

1. Answer any *six* questions : 2×6=12

(a) Prove that S_q^p is a mixed tensor of the second rank.

(b) Show that contraction of the tensor A_q^p is a scalar or invariant.

(c) Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other skew-symmetric in a pair of covariant and contravariant indices.

(d) Show that an $n \times n$ orthogonal matrix has $n(n-1)/2$ independent parameters.

(e) Show that $Su(n)$ is a group.

(f) Show that the unit element in a group is unique.

(g) If 'L' is a Lie algebra such that $\dim(L) = 3$ and $\dim(L') = 2$, then show that L' is abelian.

(h) Let G have a subgroup H and suppose that G/H is infinite cyclic. Prove that H is direct summand of G.

(i) Let $G = A \oplus B$ and let H be the subgroup containing A. Prove that $H = A \oplus (B \cap H)$.

2. Answer any *four* questions : 5×4=20

(a) If $ds^2 = g_{ij}dx^i dx^j$ is invariant, show that g_{ij} is a symmetric covariant tensor of rank 2. 5

(b) If $\phi = a_{jk}A^j A^k$ show that one can always express $\phi = b_{jk}A^j A^k$ where b_{jk} is symmetric. 5

(c) If A^μ and B_μ are any two vectors, then prove that $A^\mu B_\mu$ is invariant. 5

(d) Show that the Pauli matrices are the generators of $Su(2)$. 5

(e) Show that the rotation about *z*-axis forms a subgroup of $SO(3)$ 5

(f) If H & K be subgroups of G and $x, y \in G$ with $Hx = Ky$ then show that $H = K$. 5

(g) Write down the properties of Lie group. What is the dimensions of a Lie group ? 4+1

DSE 3 - B

(Classical Dynamics)

1. Answer any *six* questions : 2×6=12

   (a) What do you mean by 'generalised co-ordinates' of a dynamical system ?

   A particle is constrained to move on the surface of a sphere. Find the generalised co-ordinates in this case. 1+1

   (b) Define a 'cyclic coordinate'. The Lagrangian for a projectile moving under gravity is given by

   $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-mgz$, symbols being usual. Identify the cyclic coordinates, if any.

   1+1

   (c) Prove that the isotropy of space leads to the conservation of angular momentum. 2

   (d) Express Hamilton's canonical equations in terms of Poisson's brackets. 2

   (e) Show that a generalised coordinate which is ignorable in Lagrangian is also ignorable in Hamiltonian of a system. 2

   (f) Consider the following transformation :

   $Q=q\cos\alpha-p\sin\alpha$ and $P=q\sin\alpha+p\cos\alpha$,

   where q and p are the generalised position and conjugate momentum of the system respectively. Prove that the Poisson bracket $[Q,P]=1$ for any values of α. 2

   (g) What do you mean by space like and time-like four vectors ? 2

   (h) The Lagrangian of a free particle moving in a three-dimensional space is given by $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$, terms being usual. Prove that the Hamiltonian of the particle is also given by the same expression. 2

   (i) What is Reynold's number ? What is its importance in the study of fluid motion ? 1+1

2. Answer any *four* questions : 5×4=20

   (a) A simple pendulum of fixed length 'l' and bob mass 'm' is oscillating in the verticle plane. Taking the angular displacement (θ) as the generalised coordinate, show that the corresponding Lagrange's equation would be

$$\ddot{\theta}+\frac{g}{l}\theta=0$$

Also estimate the Hamiltonian of the pendulum. 4+1

(b) A particle is moving in a plane under a central attractive inverse square force. Write down its Lagrangian and derive its equation of motion.

Prove that the areal velocity of such particle remains constant. 3+2

(c) Show that Lagranges equation of motion remains unaltered if the Lagrangian $L(q, q', t)$ is added with the total time derivative $\frac{d\phi}{dt}$, where $\phi=\phi(q, t)$.

What is the familiar name of the function 'ϕ' ? 4+1

(d) The Lagrangian of a system is given by $L=\frac{1}{2}\alpha\dot{q}^2-\frac{1}{2}\delta q^2$, where α and δ are constants.

(i) Obtain the equation of motion.

(ii) Find the Hamiltonian of the system. 3+2

(e) (i) If the Lagrangian of a system does not depend on time explicitly, show its Hamiltonian remains conserved.

(ii) For a system and Hamiltonian H, show that

$\frac{dF}{dt}=\frac{\partial F}{\partial t}+[F, H]$, where $F=F(q, p, t)$ and $[F, H]$ represents a Poisson bracket. 3+2

(f) What is Doppler effect ? Derive an expression for relativistic Doppler effect using four vector concepts. 1+4

(g) Derive the expression for equation of continuity for a fluid. Hence show that for irrotational flow of incompressible fluid, the velocity potential satisfies the Laplace equation. 3+2