U.G. 4th Semester Examination 2022

PHYSICS (Honours)

Paper Code : DC - 8

Full Marks : 25 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *five* questions : 2×5=10

(a) If $x+\frac{1}{x}=2\cos\theta$, prove that $2\cos r\theta = x^r+\frac{1}{x^r}$.

(b) Find the Fourier transformation of the function $f(x)=e^{-x^2/2}$.

(c) Write down the values of the following integrals (no deduction is required) :

(i) $\int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx$ and

(ii) $\int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx$,

where $\delta(x)$ and $\delta(x-a)$ are delta functions, and $f(x)$ is a continuous function of x.

(d) Find the cube roots of –1.

(e) A box contains 10 white balls and 10 red balls. What is the probability of drawing two balls of same colour ?

(f) What do you mean by spacelike and time like four vectors ? Give one example of each kind.

(g) An observer in the *S*-frame measures the area of a circle at rest in the *xy*-plane to be 10.0 cm^2. To an observer in S'-frame moving relative to the *S*-frame with a speed of 0.8 c along the common *x*-axis, what will be the shape of the figure ? Also show that the area measured by him would be 6.0 cm^2.

2. Answer any *three* questions : 5×3=15

(a) (i) Write down the Cauchy-Riemann equations (or conditions) for a complex function $f(z) = u(x, y) + iv(x, y)$, where $z = x + iy$ and $i = \sqrt{-1}$. 2

(ii) Find the residue of

$$f(z) = \frac{e^z}{z^2 + a^2} \text{ at } z = ia\text{. Here } i = \sqrt{-1}$$ 3

(b) Expand the following function in a Fourier series :

$$f(x) = \begin{cases} -k, & \text{where } -\pi < x < 0 \\ k, & \text{where } 0 < x < \pi \end{cases}$$

Hence, show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots\ldots\ldots$. 4+1

(c) (i) Prove the 'multiplication law of probability' :

$P(A \cap B) = P(A)P(B)$, terms being usual. 3

(ii) Evaluate the integral $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the contour $|z| = \frac{1}{2}$. 2

(d) (i) If 'x' be a continuous random variable with probability density function given by

$$\begin{aligned} f(x) &= Kx, \quad (0 \le x < 2) \\ &= 2K, \quad (2 \le x < 4) \\ &= -Kx + 6K, \quad (4 \le x \le 6) \end{aligned}$$

Prove that the value of 'K' is $1/8$. 2

(ii) Two identical bodies, each of rest mass m_0, move towards each other with equal speed. They collide and stick together after the collision. Prove that in this perfectly inelastic collision, the rest mass is not conserved. Explain the result. 2+1

(e) Obtain the expression : $K = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$, where '$K$' is the kinetic energy of a particle of rest mass 'm_0' moving with a speed v, and other terms are usual.

If $\frac{v}{c} \ll 1$, prove that the above expression reduces to the familiar classical formula.

4+1
