UG/1st Sem/PHS/H/21(CBCS)

UG 1st Semester Examination 2021

PHYSICS (Honours)

Paper : DC-1

[CBCS]

Full Marks : 25 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

.

1. Answer any ***five*** questions : 2×5=10

(a) If $u\vec{a} = \vec{\nabla}v$, where u, v are scalar fields and $\vec{a}$ is a vector field, then show that

$$\vec{a}.\vec{\nabla}\times\vec{a} = 0.$$

(b) Show that the vector field $\vec{F} = 2xy\hat{\imath} + (x^2 + 2yz)\hat{\jmath} + (y^2 + 1)\hat{k}$ is conservative and find the scalar potential.

(c) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\,\hat{\imath} - 5z\,\hat{\jmath} + 10x\,\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

(d) If $\vec{A}(t)$ has a constant magnitude and $\left|\frac{d\vec{A}}{dt}\right| \neq 0$, then show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

(e) Use Stokes' theorem to show that the line integral of $\vec{F} = -y\hat{\imath} + x\hat{\jmath}$ around a closed curve in the *XY*-plane is twice the area enclosed by the curve.

(f) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

(g) Evaluate the integral $I = \int_0^{\frac{\pi}{2}} \cos t\ \delta\left(t - \frac{\pi}{4}\right) dt$.

2. Answer any ***three*** questions : 5×3=15

(a) (i)State the Taylor series for functions of one variable.

(ii) Obtain the expression for Laplace's equations in spherical polar co-ordinates for a point in space described by the co-ordinates (r, θ, φ).

(iii)What is the physical significance of $\vec{\nabla}.\vec{B} = 0$? 1+3+1=5

(b) (i)Prove that $\iiint(\phi\nabla^2\psi - \psi\nabla^2\phi)dv = \iint(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi).d\vec{s}$.

(ii) A scalar field is defined by $\phi = \phi(x, y)$. If $d\phi$ is a perfect differential then show that

$$\frac{\partial^2\phi}{\partial x\partial y} = \frac{\partial^2\phi}{\partial y\partial x}.$$

3+2=5

(c) Evaluate the integral $\oint[(xy - x^2)dx + x^2ydy]$ over the triangle bounded by the lines $y = 0, x = 1$ and $y = x$ and hence verify Green's theorem in the plane. 5

(d) Solve by the method of separation of variables:

$$3\frac{\partial U}{\partial x} + 2\frac{\partial U}{\partial y} = 0,\ where\ U(x, 0) = 4e^{-x}.$$ 5

(e) Solve the differential equation:

$$xy^2dx + (2 + x^2y)dy = 0$$

with the condition that, $y\ (at\ x = 1) = 2$. 5

UG 1st Semester Examination 2021

PHYSICS (HONOURS)

Paper : DC- 2 [CBCS]

Full Marks : 25 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

.

1. Answer any ***five*** questions : 2×5=10

(a) If a force $\vec{F}$ satisfies the relation $\vec{\nabla} \times \vec{F} = 0$, then show that the force field is conservative.

(b) A circular disc of mass M and radius r is set rolling on a horizontal table. If ω be the angular velocity of the disc, show that its total energy is $\frac{3}{4}Mr^2\omega^2$.

(c) Consider a rocket moving vertically upward against uniform gravity $\vec{g}$, that ejects gas with a constant velocity $\vec{v}$ with respect to itself. Obtain the equation of motion of the rocket.

(d) By using Euler's equations, show that the rotational kinetic energy of a rigid body is conserved when the applied torque is zero.

(e) Derive gravitational field intensity due to a point mass M at a distance r from it, using Gauss's theorem of Gravitation.

(f) Calculate the Poisson's ratio for silver, given Young's modulus for silver is 7.25×10^{10} N/m^2 and Bulk modulus is 11×10^{10} N/m^2.

(g) What is Reynold's number? How is it used to determine whether the nature of flow of a liquid is streamline or turbulent?

2. Answer any ***three*** questions : 5×3=15

(a) Prove that the total energy of a particle of mass m moving under a central force is given by

$$E = \frac{h^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where $u = \frac{1}{r}$, h = angular momentum of the particle and $V(r)$ is the potential energy at r.

5

(b) (i) The density of a sphere varies as the depth below the surface. Show that the gravitational attraction is greatest at a depth equal to $1/3$ of the radius.

(ii) What do you understand by gravitational self-energy of a homogeneous sphere?

3+2=5

(c) (i) A reference frame is rotating with an angular velocity $\vec{\omega}$ with respect to the laboratory frame. Establish the transformation relation $\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega}\times$. How does a vector transform in such cases, if it is parallel to the axis of rotation?

(ii) Calculate the principal moments of inertia of a homogeneous cube of mass M and side a.

1+1+3=5

(d) (i) Establish that shear is equivalent to elongation and contraction at right angles to each other.

(ii) A metal rod of length L and cross-section α suffers a small longitudinal strain and is stretched by l in length. Show that the potential energy stored in the rod due to this strain is $(Y\alpha l^2)/2L$ if the Young's modulus of the material is Y. 3+2=5

(e) (i) Write down the equation of continuity in case of fluid motion.

(ii) A capillary tube of radius a and length l is fitted horizontally at the bottom of a cylindrical flask of cross-section A. Initially, there is water in the flask up to a height h_1. Show that the time $T = (8\eta l A/\pi\rho g a^4)\ln\frac{h_1}{h_2}$ is required for the height to be reduced from h_1 to h_2. Here η is the coefficient of viscosity of water. 1+4=5

--o--