# 2020

# MATHEMATICS (General)

## Paper : MTMG - DC-3/GE-3

## [CBCS]

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

*Notations and symbols have their usual meanings.*

### Group - A

1. Answer any **four** questions. $1 \times 4 = 4$

(a) Evaluate

$$\int_0^{\frac{\pi}{2}} (3\sin u\, \mathbf{i} + 2\cos u\, \mathbf{j})du.$$

(b) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

(c) Write down the condition for maxima and minima for functions of two variables.

(d) Define linear span of a subset $S$ of vector space.

(e) Define directional derivative for a function $f : \mathbb{R}^2 \to \mathbb{R}$.

(f) Write down the condition for three vectors to be coplanar.

(g) If $x = r\cos\theta$, $y = r\sin\theta$, then find the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$.

## Group - B

Answer any *two* questions. 5×2=10

2. Show that the points $A$, $B$, $C$ whose position vectors are respectively $2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $4\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ form a right angled triangle. [5]

3. (a) If $\phi = 2xz^4 - x^2y$, then find $|\nabla\phi|$ at the point $(2, -2, -1)$. [2]

   (b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction $2\,\mathbf{i} - 3\,\mathbf{j} + 6\,\mathbf{k}$. [3]

4. State and prove Euler's theorem for a function of two variables. [5]

5. Find the work done in a moving particle in the force field $\mathbf{F} = 3x^2\,\mathbf{i} + (2xz - y)\,\mathbf{j} + z\,\mathbf{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [5]

## Group - C

Answer any *two* questions. 9×2=18

6. (a) Verify Stoke's theorem for $\mathbf{A} = (2x - y)\,\mathbf{i} - yz^2\,\mathbf{j} - y^2z\,\mathbf{k}$ where $S$ is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and $C$ is its boundary. [5]

   (b) Evaluate

$$\iint_R (x^2 + y^2)\,dx\,dy,$$

   where $R$ is the region bounded by $y = x^2, x = 2, y = 1$. [4]

7. (a) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$.

   [4]

   (b) Find the maximum or minimum value of $x^m y^n z^p$, subject to the condition

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1.$$

   [5]

8. (a) Verify Green's theorem in the plane for

$$\oint_C (xy + y^2)dx + x^2 dy \; ,$$

where $C$ is the closed curve of the region bounded by $y = x$ and $y = x^2$. [5]

(b) Show that $\nabla r^4 = 4r^2 \, \mathbf{r}$. [4]