

2021

MATHEMATICS (Honours)

Paper Code : VII - A & B

(New Syllabus)

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

Subject Code :

III	A	&	B
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Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

Example — If alternative A of 1 is correct, then write :

1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code :

III	A	&	B
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Subject Name :

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের রূপকে (A)/(B)/(C)/(D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি 1 নম্বর প্রশ্নের সঠিক উত্তর A হয় তবে লিখতে হবে :

1. – A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : VII - A

Full Marks : 20

Time : Thirty Minutes

Choose the correct answer.

Each question carries 2 marks.

Notations and symbols have their usual meanings.

1. Let $\vec{F} = x\vec{i} + 2y\vec{j} + 3z\vec{k}$, S be the surface of the sphere $x^2 + y^2 + z^2 = 1$ and \hat{n} be the inward unit normal vector to S . Then $\iint_S \vec{F} \cdot \hat{n} dS$ is equal to
 - A. 4π
 - B. -4π
 - C. 8π
 - D. -8π
2. Let C be the circle $x^2 + y^2 = 1$ taken in the anti-clockwise sense. Then the value of the integral $\int_C [(2xy^3 + y)dx + (3x^2y^2 + 2x)dy]$ is
 - A. 1
 - B. $\pi/2$
 - C. π
 - D. 0
3. The work done by the force $\vec{F} = 4y\vec{i} - 3xy\vec{j} + z^2\vec{k}$ in moving a particle over the circular path $x^2 + y^2 = 1, z = 0$ from $(1, 0, 0)$ to $(0, 1, 0)$ is
 - A. $\pi + 1$
 - B. $-\pi - 1$
 - C. $-\pi + 1$
 - D. $\pi - 1$

4. The number of degrees of freedom of a rigid body is
- 9
 - 3
 - 6
 - 1
5. A uniform solid cylinder rolls down along an inclined plane, inclined at an angle α with the horizon, rough enough to prevent any sliding. For pure rolling, we must have
- $\mu > \frac{2}{3} \tan \alpha$
 - $\mu > \frac{1}{2} \tan \alpha$
 - $\mu > \frac{1}{3} \tan \alpha$
 - $\mu > \frac{2}{5} \tan \alpha$
6. The minimum force P required to drag a heavy body of weight W along a rough horizontal plane is (Given: μ is the coefficient of friction, λ is the angle of friction)
- $P = W \sin \lambda$
 - $P = W \cos \lambda$
 - $P = W \tan \lambda$
 - $P = W \sec \lambda$
7. A uniform cubical box of edge a is placed on the top of a fixed sphere. Then the least radius of the sphere for which the equilibrium is stable is
- $\frac{a}{3}$
 - $\frac{a}{2}$
 - $\frac{a}{4}$
 - $\frac{a}{5}$

8. The co-ordinates (\bar{x}, \bar{y}) of c.g. of a circular arc making an angle 2α at the centre are
- A. $(\frac{a \sin \alpha}{\alpha}, 0)$
 - B. $(\frac{2}{3} \frac{a \sin \alpha}{\alpha}, 0)$
 - C. $(0, \frac{2}{3} \frac{a \cos \alpha}{\alpha})$
 - D. $(0, \frac{2}{3} \frac{a \tan \alpha}{\alpha})$
9. The moment of inertia of a hollow sphere (i.e. thin spherical shell) of mass M and radius a about any diameter is
- A. $\frac{2}{5}Ma^2$
 - B. $\frac{2}{3}Ma^2$
 - C. $\frac{1}{5}Ma^2$
 - D. $\frac{1}{3}Ma^2$
10. If K is the radius of gyration of a rigid body of mass M about an axis, then the kinetic energy of the rigid body rotating with constant angular velocity about the axis is
- A. $\frac{1}{2}MK^2\omega$
 - B. $MK^2\omega$
 - C. $MK^2\omega^2$
 - D. $\frac{1}{2}MK^2\omega^2$
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2021

MATHEMATICS (Honours)**Paper Code : VII - B****(New Syllabus)**

Full Marks : 80

Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

**Group-A
(10 Marks)**Answer any *two* questions.

1. Verify Green's theorem for

$$\int_C [(3x - 8y^2)dx + (4y - 6xy)dy],$$

where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$. 5

2. Use divergence theorem to evaluate

$$\int_S \vec{A} \cdot d\vec{S},$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ and $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$. 5

3. Evaluate the surface integral

$$\int_S (\vec{F} \cdot \vec{n})dS,$$

where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the cylinder formed by $z = 0, z = 1, x^2 + y^2 = 4$. 5

4. Verify Stokes' theorem for $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$, where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy -plane. 5

Group-B
(25 Marks)

Answer question no. 5 and any *three* from the rest.

5. Answer any *one* question 4 × 1 = 4
- (a) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1, y - x = 1, y = 2$. Find the equation of the line of action of their resultant.
- (b) A heavy elastic string whose natural length is $2\pi a$ is placed round a smooth cone whose axis is vertical and whose semi-vertical angle is α . If W is the weight and λ is the modulus of elasticity of string, prove that it will be in equilibrium when in form of a circle whose radius is $a \left(1 + \frac{W}{2\pi\lambda} \cot \alpha\right)$.
6. A solid homogeneous hemisphere rests on a rough horizontal plane and against a rough vertical wall, the coefficients of friction being μ and μ' respectively. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1} \left(\frac{3\mu}{3 + 1 + \mu\mu'} \right)$. 7
7. A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1} 3/8$, and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable. 7
8. Two equal forces act along the generators of the same system of the hyperboloid $\frac{x^2 + y^2}{a^2} - \frac{z^2}{b^2} = 1$ and cut the plane $z = 0$ at the extremities of perpendicular diameters of the circle $x^2 + y^2 = a^2$; show that the pitch of the equivalent wrench is $\frac{a^2 b}{a^2 + 2b^2}$. 7
9. Find the position of the c.g. of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ bounded by the principal planes when the density at a point (x, y, z) is kxy , where k is a constant. 7

10. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, then prove that $\tan \phi = \frac{3}{8} + \tan \theta$. 7

Group-C
(25 Marks)

Answer question no. 11 and any *three* from the rest.

11. Answer any *one* question 4 × 1 = 4
- (a) Prove that the sum of moment of inertia of a rigid body about any three perpendicular lines is constant.
- (b) Prove that the moment of momentum of a body moving in two dimensions about the origin is $Mvp + Mk^2 \frac{d\theta}{dt}$.
12. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α , show that $\omega^2 = \frac{3g}{4a \cos \alpha}$. Prove also that the direction of reaction at the hinge makes with the vertical an angle $\tan^{-1}(\frac{3}{4} \tan \alpha)$. 4+3
13. An elliptic lamina can rotate about a horizontal axis passing through a focus and perpendicular to its plane. If the eccentricity of the ellipse is $\sqrt{\frac{2}{5}}$, then show that the centre of oscillation will be at the other focus. 7
14. A circular homogeneous plate is projected up a rough inclined plane with velocity V with no rotation, the plane of the plate being in the plane of greatest slope. Show that the plate stops sliding after a time $\frac{V}{g(3\mu \cos \alpha + \sin \alpha)}$, where μ is the coefficient of friction and α is the inclination of the plane with the horizon. 7
15. Two equal uniform rods AB and AC are freely jointed at A . They are placed on a table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to AC . Show that the resulting velocities of the middle points of AB and AC are in the ratio 2:7. 7

16. A rough cylinder, of mass M , is capable of motion about its axis which is horizontal; a particle of mass m is placed on it vertically above the axis and the system is slightly disturbed. Show that the particle will slip on the cylinder when it has moved through an angle θ given by $\mu(M + 6m) \cos \theta - M \sin \theta = 4m\mu$. 7

Group-D
(20 Marks)

Answer question no. 17 and any *two* from the rest.

17. Answer any *one* question 6 × 1 = 6
- (a) Prove that the pressure at a point in a fluid in equilibrium is the same in all directions.
- (b) A circular tube is half full of liquid and is made to revolve round a vertical tangent line with angular velocity ω . If a is the radius of the tube, then prove that the diameter passing through the free surfaces of the liquid is inclined at an angle $\tan^{-1}(\frac{\omega^2 a}{g})$ to the horizon.
18. A closed right circular cylinder is very nearly filled with water and is made to rotate about its axis which is vertical. If the angular velocity is $\frac{\sqrt{2gh}}{a}$, then show that the whole thrust on the base is half as much again when the liquid is at rest, where h is the height and a is the radius of the cylinder. 7
19. A vertical circular cylinder of height $2h$ and radius r , closed at the top, is just filled by equal volumes of two liquids of densities ρ and σ , ($\sigma > \rho$). If the axis gradually inclined to the vertical, then show that the pressure at the lowest point of the base will never exceed $g(\rho + \sigma)(r^2 + h^2)^{1/2}$. 7
20. A semi-circular lamina is completely immersed in water with its plane vertical, so that the extremity A of its bounding diameter is in the surface, and the diameter makes with its surface an angle α . If E is the centre of pressure and ϕ is the angle between AE and the diameter, then prove that $\tan \phi = \frac{3\pi + 16 \tan \alpha}{16 + 15\pi \tan \alpha}$. 7

21. A cylindrical piece of wood of length l and sectional area α is floating with its axis vertical in a cylindrical vessel of sectional area A which contains water. Prove that the work done in slowly pressing down the wood until it is completely immersed is $\frac{1}{2}gl^2\alpha\left(1 - \frac{\alpha}{A}\right)\frac{(\rho - \sigma)^2}{\rho}$, where ρ and σ are the densities of water and wood respectively. 7
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