UG/4th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-9

(Mechanics)

Full Marks : 32

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

(a) A particle describes a parabola with uniform speed. Show that its angular velocity about the focus S, at any point P, varies inversely as $(SP)^{3/2}$.

(b) Prove that the potential V at a point distant R from the centre of the earth is given by $-\dfrac{GM}{R}$.

(c) State the Varignon's theorem.

(d) For a rectilinear motion of a particle if an impulse I changes its velocity from u to v and E is the change of kinetic energy, show that $E = I\left(\dfrac{u+v}{2}\right)$.

P.T.O.

(e) Write down the expression for *virial* and give its significance.

(f) Obtain the equation of the line of action of the resultant force of a system of coplanar forces.

(g) Find the c.g. of the semi-circle.

Group - B

(10 Marks)

Answer any *two* questions. 5×2=10

2. A particle of mass m is acted on by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 5

3. If a planet were suddenly stopped in its orbit, supposed circular, show that it would fall into the sum in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 5

4. Three forces P, Q, R act along the sides of a triangle formed by the lines $x+y=3$, $2x+y=1$ and $x-y+1=0$. Find the equation of the line of action of the resultant. 5

5. Four equal rods each of weight W form a rhombus $ABCD$ with, smooth hinges at the joints. The frame is suspended by the end A and a weight W' is attached at C. A stiffening rod of negligible weight joins the middle points of AB and AD keeping these inclined at an angle α to AC. Show that the thrust in the stiffening rod is $(4W + 2W')\tan\alpha$. 5

Group - C

(18 Marks)

Answer any *two* questions. 9×2=18

6. (a) The middle points of opposite sides of a quadrilateral formed by four freely jointed weightless bars are connected by two light rods of length a and b in a state of tension. If T_1 and T_2 be the tensors of those rods, prove that $\frac{T_1}{a} + \frac{T_2}{b} = 0$. 5

(b) A circular orbit of radius a is described under central attractive force

$$f(r) = \mu\left[\frac{b}{r^2} + \frac{c}{r^4}\right];\ \mu > 0.$$

Prove that the motion is stable if $a^2b - c > 0$. 4

7. (a) Forces X, Y, Z act along the three straight lines $y = b,\ z = -c;\ z = c,\ x = -a;\ x = a,\ y = -b$

respectively; show that they will have a single resultant if $\frac{a}{X}+\frac{b}{Y}+\frac{c}{Z}=0$ and that the equations of its line of action are any two of the three

$$\frac{y}{Y}-\frac{z}{Z}-\frac{a}{X}=0,\ \frac{z}{Z}-\frac{x}{X}-\frac{b}{Y}=0,\ \frac{x}{X}-\frac{y}{Y}-\frac{c}{Z}=0.$$

6

(b) For motion from rest under the attraction following inverse square of the distance if V be the velocity acquired by the particle in falling from rest at infinity to a distance R, prove that $V^2=\frac{2\mu}{R}$. 3

8. (a) If a particle be projected from an apse at a distance a with a velocity from infinity under the action of a central force μr^{-2n-3}, then prove that the path is $r^n=a^n\cos n\theta$. 5

(b) The algebraic sums of the moments of a system of coplanar forces about the points whose co-ordinates are (1, 0), (0, 2) and (2, 3) referred to rectangular axes are H, $2H$ and $3H$ respectively. Find the tangent of the angle which the direction of the resultant force makes with the axis of x. 4

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-8

(Differential Equations)

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Determine degree and order of the differential equation

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{2/3}=\frac{d^2y}{dx^2}$$

(b) Find the complete solution of

$$y=px+p^2,\text{ where } p=\frac{dy}{dx}.$$

(c) Find the complementary function of the differential equation

$$\left(D^4-2D^3+D^2\right)y=x^3$$

P.T.O.

(d) Define ordinary point of a second order linear homogeneous differential equation. 1

(e) Define first order quasi-linear partial differential equation. 1

(f) Verify that origin is a regular singular point of $2x^2y'' + xy' - (x+1)y = 0$. 1

(g) Obtain PDE from $z = f(\sin x + \cos y)$. 1

Group - B

(10 Marks)

Answer any *two* questions. 5×2=10

2. Prove that $(x+y+1)^{-4}$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ and solve it. 5

3. Find the power series solution of the equation

$$\frac{d^2y}{dx^2} + (x-1)^2\frac{dy}{dx} - 4(x-1)y = 0$$

about the point $x = 1$. 5

4. Solve $q = (z + px)^2$ using Charpit's method. 5

5. Write down the recurrence relation of Legendre polynomials and hence obtain $P_5(x)$. 5

Group - C

(18 Marks)

Answer any *two* questions. $9\times2=18$

6. (a) Solve $(x+3)^2\frac{d^2y}{dx^2}-4(x+3)\frac{dy}{dx}+6y=x$. 4

(b) Using the method of variation of parameters solve

$$\frac{d^2y}{dx^2}+a^2y=\sec ax.$$ 5

7. (a) Solve by the method of undetermined coefficients the differential equation

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=24e^{-3x}.$$ 4

(b) Find the eigen values and eigenfunctions of the boundary value problem

$$\frac{d^2y}{dx^2}+\lambda y=0$$

with $y(0)=0$ and $y'(\pi)=0$. 5

8. (a) Find a complete integral of $(p+q)(px+qy)=1$. 6

(b) Prove that $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$. 3

UG/4th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-10

(Probability and Statistics)

Full Marks : 32

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) In a Binomial distribution with Parameters $(8, P)$, $P(X=2)=P(X=3)$, find $P(X=0)$ and $P(X=8)$.

(b) Show that the function $f(x)$ is given by

$$f(x)=\begin{cases}|x|, & -1<x<1 \\ 0, & \text{elsewhere}\end{cases}$$

is a possible probability density function.

(c) If a random variable X takes the values

$$x_K=\frac{(-1)^K 2^K}{K}, \quad K=1, 2, ...$$

with probabilities $P_K=\frac{1}{2^K}$, find $E(X)$.

P.T.O.

(d) A die is thrown 108 times in Succession. Find the expectation and variance of the number of 'Six' appeared. 1

(e) A point is taken at random within the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a<b)$. What is the chance that its distance from the centre exceeds a?

$\int_0^m \lambda e^{-\lambda x}\,dx$

(f) Find the median of the distribution

$$f(x)=\lambda e^{-\lambda x},\ (\lambda>0),\ x>0$$

$-\lambda\left[\frac{e^{-\lambda x}}{\lambda}\right]_0^m$

$(e^{-m\lambda}-1)$

(g) Let $(x_1, x_2, \ldots, x_n)$ be a random sample from a normal population $N(\mu, 1)$. Show that $t=\frac{1}{n}\sum_{i=1}^{n}x_i^2$ is an unbiased estimate of μ^2+1.

Group - B

(10 Marks)

Answer any *two* questions. 5×2=10

2. Let X be a continuous random variable with probability density function given by

$$f(x)=\begin{cases} Kx, & 0\le x\le 1 \\ K, & 1\le x\le 2 \\ -Kx+3K & 2\le x\le 3 \\ 0\ , & x>3 \end{cases}$$

Determine K and find $F(x)$. 5

3. If $(a \neq 0)$, $c(\neq 0)$, b, d are Constants, prove that $\rho(aX+b, cY+d) = \frac{ac}{|a||c|}\rho(X,Y)$. 5 5

4. A point P is taken at random in a line AB of length $2a$. Find the mathematical expectation of $AP.PB$ and that of the difference $|AP - PB|$. Also find the probability that the area of rectangle having sides AP and BP will exceed $\frac{1}{2}a^2$. 5

5. A random variable X has probability density function $12x^2(1-x)$, $(0 < x < 1)$. Compute $P(|X-m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality. 5

5

Group - C

(18 Marks)

Answer any *two* questions. 9×2=18

6. (a) The joint probability density function of the random variable X and Y is

$$f(x, y) = K(1-x-y), \text{ for } x \geq 0, y \geq 0, x+y \leq 1$$
$$= 0, \quad \text{elsewhere}$$

where K is a constant.

Find (i) The marginal probability density function

P.T.O.

(ii) The mean value of y when $x = \frac{1}{2}$

(iii) The covariance of x and y. 6

(b) Five balls are drawn from an urn containing 3 white balls and 7 black balls. Find the probability distribution of the number of white balls drawn without replacement. 3 6+3

7. (a) Prove that for the binomial population with probability mass function :

$$f(x, p) = {}^nC_x p^x q^{n-x}, \quad x = 1, 2, \ldots, n$$
$$q = 1 - p$$ 5

the maximum Likelihood estimator of p is $\frac{x}{n}$. Also find its variance.

(b) X and Y are two random variables each having the standard deviation unity. The correction co-efficient between $aX + bY$ and $bX + aY$ is $\frac{1+2ab}{a^2+b^2}$. Find the co-rrelation co-efficient between X and Y. 5+4

4

8. (a) The fraction of defective items in a large lot is P. To test the null hypothesis $H_0 : P = 0.2$, one considered the number 'f' of defectives in a sample of 8 items and accepts the hypothesis if $f \leq 6$, and rejects the hypothesis otherwise. What is the probability of type-I error of this test? What is the

probability of type-II error corresponding to $P = 0.1$?

(b) A continuous distribution is given by the density function

$$f(x) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{1}{2}(\log x)^2} \quad \text{for } x > 0$$
$$= 0 \quad , \quad \text{for } x < 0$$

find mean, mode and standard deviation of the distribution.

4+5