

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH SEC-1

(Discrete Mathematics)

Full Marks : 32

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4
- (a) Give an example of a multigraph.
 - (b) Show that if in a graph G there is one and only one path between every pair of vertices, then G is a tree.
 - (c) Find the complement of the Boolean function $f = (a+b)(a+c')$.
 - (d) Let A, B be subsets of a universal set. Prove that $A=B$ iff $A\Delta B = \phi$, where $A\Delta B = (A - B) \cup (B - A)$.

P.T.O.

(2)

(e) In a group of 6 people, prove that there are three mutual friends or three mutual strangers.

(f) Show that

$$p \rightarrow q \equiv \neg q \rightarrow \neg p.$$

(g) Let $S = \{a, b, c\}$. Consider the poset $P(S)$ with the partial order as \subseteq . Verify that $(P(S), \cup, \cap)$ is a distributive lattice.

Group - B

(10 Marks)

Answer any *two* questions : $5 \times 2 = 10$

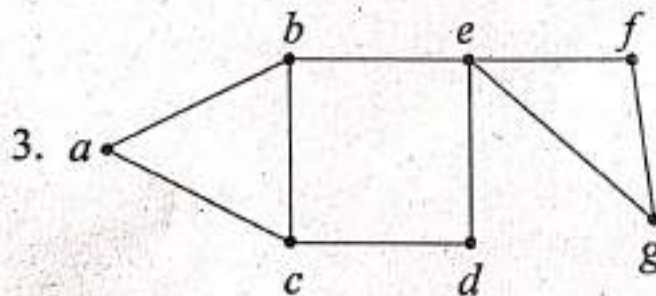
2. Let $A = \{1, 2, 3, 4\}$. Give example of a relation in A which is —

(i) partial order relation,

(ii) reflexive, transitive but not symmetric,

(iii) transitive but not reflexive or symmetric.

$2 + 1\frac{1}{2} + 1\frac{1}{2}$

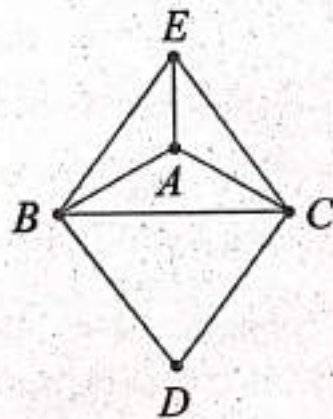


For the above graph, determine the following :

(3)

- (a) a path from b to d ,
- (b) a closed walk from b to d that is not a circuit,
- (c) a circuit from b to b that is not a cycle. 1+2+2

4. Consider the following graph.



- (a) Is it Hamiltonian? Justify.
- (b) Is there a Hamiltonian Path? Justify.
- (c) Is it Eulerian? Justify.
- (d) Is there an Eulerian trail? Justify. 1+2+1+1

5. Let p , q and r be primitive statements. Verify that each of the following is a tautology or not.

(i) $[p \vee (q \wedge r)] \vee \neg [p \vee (q \wedge r)]$

(ii) $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \neg(p \vee q)]$ 2+3

P.T.O.

(4)

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) Find the K-map for the following expression :

$$x'y'zw + x'yzw' + xy'zw + xyzw'. \quad 4$$

- (b) Express the Boolean function $f(x, y, z) = (x + y + z)(xy + xz)$; $x, y, z \in B$ in disjunction normal form (DNF). 5

7. (a) Identify the bound variables and the free variables in each of the following expressions.

(i) $\forall y \exists z [\cos(x + y) = \sin(z - x)]$

(ii) $\exists x \exists y [x^2 - y^2 = z]$

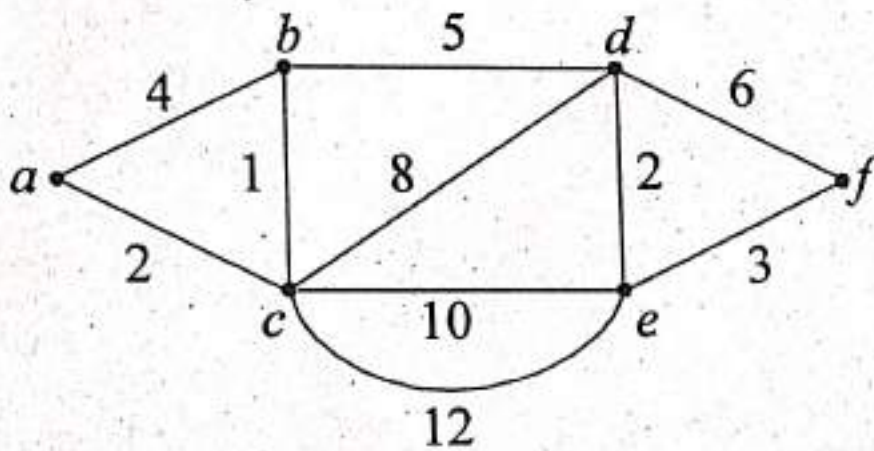
In both cases the universe comprises all real numbers. 2+2

- (b) Find the solution to the recurrence relation, $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$, $a_0 = 3$, $a_1 = 7$. 5

8. (a) Find the maximum and minimum number of edges of a simple graph with 12 vertices and 4 components. 4

(5)

(b) Using Dijkstra's algorithm to find the length of the shortest path of the following graph from the vertex a to f :



5

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-12

(Numerical Methods and C Programming Language)

Full Marks : 32

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

[Examinees are allowed to use Non-Programmable
Calculator for Numerical Calculation.]

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

- (a) How many significant digits does the floating point number 0.03140×10^3 have?
- (b) Prove that $(1 + \Delta)(1 - \nabla) = 1$.
- (c) State the conditions under which Newton-Raphson method fails to solve a polynomial equation.
- (d) Evaluate $\int_0^1 x^2 dx$ by Trapezoidal Rule taking 5 sub-intervals and find the relative error in the result.

P.T.O.

(2)

- (e) Write the error term of Simpson's 1/3 rule.
- (f) What is the difference between $abs()$ and $fabs()$ functions?
- (g) What are the derived data types in C-programming language?

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

- 2. Using Newton-Raphson method, find the value of $\sqrt[5]{2}$, correct upto 5 significant figures. 5
- 3. Derive the total number of operations required in Gauss-Elimination method. 5
- 4. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 3/8 rule. 5
- 5. Using Lagrange's interpolation formula, find the value of $f(x)$ for $x = 1$ from the following table :

x	-1	0	2	5
$f(x)$	9	5	3	15

5

(3)

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) Derive Newton's Forward Interpolation formula.
State when it can be used. 7
- (b) Find the value of $\left(\frac{\Delta}{E}\right) \sin(2x)$. 2
7. (a) Using 4th order Runge-Kutta method, find an
approximate value of y for $x = 0.2$ if $\frac{dy}{dx} = x + y^2$,
given that $y = 1$ when $x = 0$ and $h = 0.1$. 5
- (b) Define rate of convergence. Determine the rate of
convergence for the Secant method. 4
8. (a) Write a program in C to find the value of $n!$. 4
- (b) What are the differences between 'while' and 'if'
statements in C-programming? 3
- (c) Write a short note on control-string in C-
programming. 2
-

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-11

(Advanced Analysis on \mathbb{R} & \mathbb{C})

Full Marks : 32

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

- (a) Is it possible to define a metric on any non-empty set? Justify your answer.
- (b) If f be an analytic function on a region $G(\subset \mathbb{C})$ such that $\text{Im}f = 0$, then show that f is constant.
- (c) Find the Laurent series expansion of the function
$$\frac{7z-2}{z(z-2)(z+1)}$$
 in the domain $|z| > 2$.
- (d) Describe explicitly, the ball $\mathcal{B}(x_0, r)$ in the metric space (\mathbb{R}, d) , where $d(x, y) = |x - y|$.

P.T.O.

(2)

- (e) Let $d(x, y) = |x^2 - y^2|$, $x, y \in \mathbb{R}$. Verify d is a metric on \mathbb{R} or not.
- (f) Examine the analyticity of the function $f(z) = |z-1|^2$ at $z = 1, 2$.
- (g) Show that the subset $A = [0, 1)$ of the metric space (X, d) where $X = [0, 2)$ and ' d ' is the usual metric is an open set.

Group - B

(10 Marks)

Answer any *two* questions : $5 \times 2 = 10$

2. Let $X = l_p$ ($1 \leq p < \infty$), set of all p th summable sequences of real or complex numbers and let

$$d(x, y) = \left\{ \sum_{i=1}^{\infty} |x_i - y_i|^p \right\}^{\frac{1}{p}} \quad \text{where } x = \{x_n\} \text{ and}$$

$y = \{y_n\} \in l_p$. Show that ' d ' is a metric on $X = l_p$. 5

3. Prove that for a complete metric space (X, d) , every nested sequence of non-empty closed sets $\{F_n\}$ with $\text{diam}(F_n) \rightarrow 0$ as $n \rightarrow \infty$, has a non-empty intersection containing precisely one point. 5

4. (a) Suppose $f(z) = u(x, y) + iv(x, y)$ and $\overline{f(z)}$

(3)

are both analytic throughout their domain D . Show that $f(z)$ must be constant throughout D . 3

(b) Show that the function $f(z) = |z|^2$ is nowhere differentiable except at the origin. 2

5. Show that the function $u(x, y) = \frac{y}{x^2 + y^2}$ is harmonic in some domain and find σ a harmonic conjugate of $u(x, y)$. 5

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Let $X = \{1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots\}$ and ' d ' is the usual metric defined on X . Let

$$A = \{1, \frac{1}{3}, \frac{1}{5}, \dots, \frac{1}{2n-1}, \dots\} \text{ and}$$

$$B = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots, \frac{1}{2n}, \dots\}.$$

Find the distance between A and B . 3

(b) Evaluate $\int_C \frac{\cosh(\pi z)}{z(z^2 + 1)} dz$ using Cauchy's integral

formula when $C: |z| = 2$. 3

(c) Prove that every compact metric space is separable. 3

P.T.O.

7. (a) Consider any non-empty set X together with the metric

$$d(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

Examine whether (X, d) is complete or not. 3

- (b) Let (X, d) be a metric space and \mathcal{A} be a non-empty subset of X . Then show that (X, d) is connected if and only if every continuous mapping $f: \mathcal{A} \rightarrow \{0, 1\}$ is a constant mapping. 3

- (c) Find the value of the integral

$$\int_0^{1+i} (x - y + ix^2) dx$$

along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. 3

8. (a) Let (X, d) and (Y, d') be two metric spaces. Show that a function $f: X \rightarrow Y$ is continuous if and only if for any $x \in X$ and for all sequence $\{x_n\}_{n \in \mathbb{N}}$ converges to x in (X, d) , the sequence $\{f(x_n)\}_{n \in \mathbb{N}}$ converges to $f(x)$ in (Y, d') . 6

- (b) Show that $\int_C (z - z_0)^n dz = \begin{cases} 2\pi i, & \text{if } n = -1 \\ 0 & \text{if } n \neq -1 \end{cases}$

where C is the circle with centre z_0 and radius $r > 0$ traversed in the anti-clockwise direction. 3

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DSE-2A/2B/2C

Full Marks : 32

Time : Two Hours

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in their own words as far as practicable.*

DSE 2A

(Differential Geometry)

Group - A

(4 Marks)

1. Answer any *four* questions :

1×4=4

(a) Calculate $g = |g_{ij}|$ when

$$ds^2 = (dx)^2 + 2\cos\phi dx dy + (dy)^2.$$

(b) Let $I \subset \mathbb{R}$ be an open interval and let $x, \tau: I \rightarrow \mathbb{R}$ be two differentiable functions such that $x(s) > 0$ for all $s \in I$. Does there exist a curve in \mathbb{R}^3 with x as its curvature and τ as its torsion?

P.T.O.

- (c) For which condition a space curve (i.e., a curve in \mathbb{R}^3) reduces into a plane curve (i.e., a curve lying on a plane)?
- (d) Does the first fundamental form of a surface in \mathbb{R}^3 depend on the parametrization of the surface?
- (e) Mention true or false : Any surface in \mathbb{R}^3 has an empty interior when thought of as a subset of \mathbb{R}^3 .
- (f) Find the curvature of the space curve $r(t) = (a, bt, c/t)$ at $t = 1$.
- (g) Does the Weingarten map for a surface in \mathbb{R}^3 change sign when the orientation of the surface changes?

Group - B

(10 Marks)

Answer any *two* questions : $5 \times 2 = 10$

2. Calculate the components of Riemann tensor for the metric

$$ds^2 = dx^2 + f(x, y)dy^2,$$

where f is a smooth function of x and y .

3. State and prove the Serret-Frenet formulae for a regular curve in \mathbb{R}^3 .

4. Calculate the first and second fundamental form for the surface of revolution

$$\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u)),$$

where f and g are continuously differentiable functions.

5. Let S be a regular surface in \mathbb{R}^3 . Prove that S is orientable if and only if there exists a continuous function $N:S \rightarrow S^2$, where S^2 is the unit sphere, such that $N(p)$ is normal to S at p for each $p \in S$.

Group - C

(18 Marks)

Answer any two questions : $9 \times 2 = 18$

6. (a) Let $\gamma:I \rightarrow \mathbb{R}^3$ be a curve in \mathbb{R}^3 parametrized by arc length. Prove that γ is a straight line or a segment of a straight line in \mathbb{R}^3 if and only if its curvature vanishes everywhere. 4

- (b) Consider as a surface S the hyperbolic paraboloid given by

$$S = \{(x, y, z) \in \mathbb{R}^3 : z = x^2 - y^2\}.$$

Study the second fundamental form of S at $(0, 0, 0)$ and show that its Gauss curvature at $(0, 0, 0)$ is negative. 5

P.T.O.

7. (a) Show that all contravariant transformations form a group under suitable composition. 5
- (b) Prove that for any two surfaces S_1 and S_2 in \mathbb{R}^3 , a local diffeomorphism $f:S_1 \rightarrow S_2$ is a local isometry if for any patch σ of S_1 , the patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2 , respectively, have the same first fundamental form. 4
8. (a) Calculate the Frenet Apparatus $\{T, N, B, \kappa, \tau\}$ of the curve γ in \mathbb{R}^3 given by
$$\gamma(t) = (t - \sin t \cos t, \sin^2 t, \cos t), t \in (0, \pi). \quad 5$$
- (b) Prove that any tangent developable is locally isometric to a plane. 4
-

(5)

DSE 2B

(Fluid Mechanics)

Group - A

(4 Marks)

1. Answer any *four* questions :

1×4=4

- (a) If the velocity distribution is $u = iAx^2y + jBy^2zt + kCzt^2$, where A, B, C are constants, then find the acceleration and velocity components.
- (b) Why is the centre of pressure is below the centre of gravity for an incline surface?
- (c) Define steady and unsteady flow with examples.
- (d) Determine the acceleration at the point (2, 1, 3) at $t = 0.5$ sec, if $u = yz + t$, $v = xz - t$ and $w = xy$.
- (e) Show that there exist surfaces which cut streamline orthogonally if the velocity potential exist.
- (f) Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.
- (g) Show that free surface of a heavy homogeneous liquid at rest under gravity is horizontal.

P.T.O.

Group - B

(10 Marks)

Answer any two questions :

5×2=10

2. In the two-dimensional motion of a liquid, if the current coordinates (x, y) are expressible in the forms of initial coordinate (a, b) and the time, then show that the motion is irrotational if

$$\frac{\partial(\dot{x}, x)}{\partial(a, b)} + \frac{\partial(\dot{y}, y)}{\partial(a, b)} = 0. \quad 5$$

3. A mass of fluid is in motion so that the line of motion lie on the surface of co-axial cylinders. Show that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho u) + \frac{\partial}{\partial z} (\rho v) = 0$$

where u, v are the velocity perpendicular and parallel to z . 5

4. Prove that if the forces per unit mass at the point (x, y, z) parallel to the axes are $y(a-z), x(a-z), xy$; then the surface of equal pressure represents a hyperbolic paraboloid. 5

5. A semi-circular area is completely immersed in water with its plane vertical, so that the extremity A of its bounding diameter is in surface and the diameter makes

3/2

Ans

$\frac{2 \left(\frac{5}{2} \right) \left(\frac{3}{2} \right)}{2 \left(\frac{3}{2} \right) \left(\frac{3}{2} \right)}$

$\frac{5/2 \times 3/2 = 4}{2 \times 3/2 = 3}$

$\frac{4}{3} \times \left(\frac{3}{2} \right) = 2$

$\frac{3}{2} \times \left(\frac{3}{2} \right) = \frac{9}{4}$

$\frac{9}{4} \times \left(\frac{3}{2} \right) = \frac{27}{8}$

$\frac{27}{8} \times \left(\frac{3}{2} \right) = \frac{81}{16}$

$\frac{81}{16} \times \left(\frac{3}{2} \right) = \frac{243}{32}$

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$\frac{729}{64} \times \left(\frac{3}{2} \right) = \frac{2187}{128}$

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$\frac{6561}{256} \times \left(\frac{3}{2} \right) = \frac{19683}{512}$

$\frac{19683}{512} \times \left(\frac{3}{2} \right) = \frac{59049}{1024}$

$\frac{59049}{1024} \times \left(\frac{3}{2} \right) = \frac{177147}{2048}$

$\frac{177147}{2048} \times \left(\frac{3}{2} \right) = \frac{531441}{4096}$

$\frac{531441}{4096} \times \left(\frac{3}{2} \right) = \frac{1594323}{8192}$

$\frac{1594323}{8192} \times \left(\frac{3}{2} \right) = \frac{4782969}{16384}$

$\frac{4782969}{16384} \times \left(\frac{3}{2} \right) = \frac{14348907}{32768}$

$\frac{14348907}{32768} \times \left(\frac{3}{2} \right) = \frac{43046721}{65536}$

$\frac{43046721}{65536} \times \left(\frac{3}{2} \right) = \frac{129140163}{131072}$

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$\frac{282429536481}{16777216} \times \left(\frac{3}{2} \right) = \frac{847288609443}{33554432}$

$\frac{847288609443}{33554432} \times \left(\frac{3}{2} \right) = \frac{2541865828329}{67108864}$

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$\frac{7625597484987}{134217728} \times \left(\frac{3}{2} \right) = \frac{22876792454961}{268435456}$

$\frac{22876792454961}{268435456} \times \left(\frac{3}{2} \right) = \frac{68630377364883}{536870912}$

$\frac{68630377364883}{536870912} \times \left(\frac{3}{2} \right) = \frac{205891132094649}{1073741824}$

$\frac{205891132094649}{1073741824} \times \left(\frac{3}{2} \right) = \frac{617673396283947}{2147483648}$

$\frac{617673396283947}{2147483648} \times \left(\frac{3}{2} \right) = \frac{1853020188851841}{4294967296}$

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$\frac{5559060566555523}{8589934592} \times \left(\frac{3}{2} \right) = \frac{16677181699666569}{17179869184}$

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$\frac{150094635296999121}{68719476736} \times \left(\frac{3}{2} \right) = \frac{450283905890997363}{137438953472}$

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$\frac{984770902183611232881}{17592186044416} \times \left(\frac{3}{2} \right) = \frac{2954312706550833698643}{35184372088832}$

$\frac{2954312706550833698643}{35184372088832} \times \left(\frac{3}{2} \right) = \frac{8862938119652501095929}{70368744177664}$

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$\frac{58149737003040059780390169}{18014398509481984} \times \left(\frac{3}{2} \right) = \frac{174449211009120179341170507}{36028797018963968}$

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$\frac{4710128697246244842211603689}{288230376151711744} \times \left(\frac{3}{2} \right) = \frac{14130386091738734526634811067}{576460752303423488}$

$\frac{14130386091738734526634811067}{576460752303423488} \times \left(\frac{3}{2} \right) = \frac{42391158275216203579904433201}{1152921504606846976}$

$\frac{42391158275216203579904433201}{1152921504606846976} \times \left(\frac{3}{2} \right) = \frac{127173474825648610739713299603}{2305843009213693952}$

$\frac{127173474825648610739713299603}{2305843009213693952} \times \left(\frac{3}{2} \right) = \frac{381520424476945832219139898809}{4611686018427387904}$

$\frac{381520424476945832219139898809}{4611686018427387904} \times \left(\frac{3}{2} \right) = \frac{1144561273430837496657419696427}{9223372036854775808}$

$\frac{1144561273430837496657419696427}{9223372036854775808} \times \left(\frac{3}{2} \right) = \frac{3433683820292512489972259089281}{18446744073709551616}$

$\frac{3433683820292512489972259089281}{18446744073709551616} \times \left(\frac{3}{2} \right) = \frac{10301051460877537469916777267843}{36893488147419103232}$

$\frac{10301051460877537469916777267843}{36893488147419103232} \times \left(\frac{3}{2} \right) = \frac{30903154382632612409750331803529}{73786976294838206464}$

$\frac{30903154382632612409750331803529}{73786976294838206464} \times \left(\frac{3}{2} \right) = \frac{92709463147897837229250995410587}{147573952589676412928}$

$\frac{92709463147897837229250995410587}{147573952589676412928} \times \left(\frac{3}{2} \right) = \frac{278128389443693511687752986231761}{295147905179352825856}$

$\frac{278128389443693511687752986231761}{295147905179352825856} \times \left(\frac{3}{2} \right) = \frac{834385168331080535063258958695283}{590295810358705651712}$

$\frac{834385168331080535063258958695283}{590295810358705651712} \times \left(\frac{3}{2} \right) = \frac{2503155504993241605189776876085849}{1180591620717411303424}$

$\frac{2503155504993241605189776876085849}{1180591620717411303424} \times \left(\frac{3}{2} \right) = \frac{7509466514979724815569330628257547}{2361183241434822606848}$

$\frac{7509466514979724815569330628257547}{2361183241434822606848} \times \left(\frac{3}{2} \right) = \frac{22528399544939174446707991884772641}{4722366482869645213696}$

$\frac{22528399544939174446707991884772641}{4722366482869645213696} \times \left(\frac{3}{2} \right) = \frac{67585198634817523340123975654317923}{9444732965739290427392}$

$\frac{67585198634817523340123975654317923}{9444732965739290427392} \times \left(\frac{3}{2} \right) = \frac{202755595904452570020371926962953769}{18889465931478580854784}$

$\frac{202755595904452570020371926962953769}{18889465931478580854784} \times \left(\frac{3}{2} \right) = \frac{608266787713357710061115780888861307}{37778931862957161709568}$

$\frac{608266787713357710061115780888861307}{37778931862957161709568} \times \left(\frac{3}{2} \right) = \frac{1824800363140073130183347342666583921}{75557863725914323419136}$

$\frac{1824800363140073130183347342666583921}{75557863725914323419136} \times \left(\frac{3}{2} \right) = \frac{5474401089420219390549042028000751763}{151115727451828646838272}$

$\frac{5474401089420219390549042028000751763}{151115727451828646838272} \times \left(\frac{3}{2} \right) = \frac{16423203268260658171647126084002255289}{302231454903657293676544}$

$\frac{16423203268260658171647126084002255289}{302231454903657293676544} \times \left(\frac{3}{2} \right) = \frac{49269609804781974514941378252006765867}{604462909807314587353088}$

$\frac{49269609804781974514941378252006765867}{604462909807314587353088} \times \left(\frac{3}{2} \right) = \frac{147808829414345923544824134756020297601}{1208925819614629174706176}$

$\frac{147808829414345923544824134756020297601}{1208925819614629174706176} \times \left(\frac{3}{2} \right) = \frac{443426488243037770634472404268060892803}{2417851639229258349412352}$

$\frac{443426488243037770634472404268060892803}{2417851639229258349412352} \times \left(\frac{3}{2} \right) = \frac{1330279464729113311903417212804182678409}{4835703278458516698824704}$

$\frac{1330279464729113311903417212804182678409}{4835703278458516698824704} \times \left(\frac{3}{2} \right) = \frac{3990838394187339935710251638412548035227}{9671406556917033397649408}$

$\frac{3990838394187339935710251638412548035227}{9671406556917033397649408} \times \left(\frac{3}{2} \right) = \frac{11972515182562019807130754915237644105641}{19342813113834066795298816}$

$\frac{11972515182562019807130754915237644105641}{19342813113834066795298816} \times \left(\frac{3}{2} \right) = \frac{35917545547686059421392264745712932316923}{38685626227668133590597632}$

$\frac{35917545547686059421392264745712932316923}{38685626227668133590597632} \times \left(\frac{3}{2} \right) = \frac{107752636643058178264176794237138796950769}{77371252455336267181195264}$

$\frac{107752636643058178264176794237138796950769}{77371252455336267181195264} \times \left(\frac{3}{2} \right) = \frac{323257909929174534792530382711416390852307}{154742504910672534362390528}$

$\frac{323257909929174534792530382711416390852307}{154742504910672534362390528} \times \left(\frac{3}{2} \right) = \frac{969773729787523604377591148134249172556921}{309485009821345068724781056}$

$\frac{969773729787523604377591148134249172556921}{309485009821345068724781056} \times \left(\frac{3}{2} \right) = \frac{2909321189362570813132773444402747517670763}{618970019642690137449562112}$

$\frac{2909321189362570813132773444402747517670763}{618970019642690137449562112} \times \left(\frac{3}{2} \right) = \frac{8727963568087712439398320333208242553012289}{1237940039285380274899124224}$

$\frac{8727963568087712439398320333208242553012289}{1237940039285380274899124224} \times \left(\frac{3}{2} \right) = \frac{26183890704263137318194960999624727659036867}{2475880078570760549798248448}$

$\frac{26183890704263137318194960999624727659036867}{2475880078570760549798248448} \times \left(\frac{3}{2} \right) = \frac{78551672112789411954584882998874182977110601}{4951760157141521099596496896}$

$\frac{78551672112789411954584882998874182977110601}{4951760157141521099596496896} \times \left(\frac{3}{2} \right) = \frac{235655016338368235363754648996622548931331803}{9903520314283042199192993792}$

$\frac{235655016338368235363754648996622548931331803}{9903520314283042199192993792} \times \left(\frac{3}{2} \right) = \frac{706965049015104706091263946989867646793995407}{19807040628566084398385987584}$

$\frac{706965049015104706091263946989867646793995407}{19807040628566084398385987584} \times \left(\frac{3}{2} \right) = \frac{2120895147045314118273791840969602940381986221}{39614081257132168796771975168}$

$\frac{2120895147045314118273791840969602940381986221}{39614081257132168796771975168} \times \left(\frac{3}{2} \right) = \frac{6362685441135942354821375522908808821145958663}{79228162514264337593543950336}$

$\frac{6362685441135942354821375522908808821145958663}{79228162514264337593543950336} \times \left(\frac{3}{2} \right) = \frac{19088056323407827064464126568726426463437875989}{158456325028528675187087900672}$

$\frac{19088056323407827064464126568726426463437875989}{158456325028528675187087900672} \times \left(\frac{3}{2} \right) = \frac{57264168970223481193392379706179279390313627967}{316912650057057350374175801344}$

$\frac{57264168970223481193392379706179279390313627967}{316912650057057350374175801344} \times \left(\frac{3}{2} \right) = \frac{171792506910670443580177139118537838170940883901}{633825300114114700748351602688}$

$\frac{171792506910670443580177139118537838170940883901}{633825300114114700748351602688} \times \left(\frac{3}{2} \right) = \frac{515377520732011330740531417355613514512822651703}{1267650600228229401496703205376}$

$\frac{515377520732011330740531417355613514512822651703}{126$

(7)

with the surface an angle α . Prove that if E be the C.P. and θ the angle between AE and the diameter,

$$\tan \theta = \frac{3\pi + 16 \tan \alpha}{16 + 15\pi \tan \alpha} \quad 5$$

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) A semi-circular tube has its bounding diameter horizontal and contains equal volumes of n fluids of densities successively equal to $\rho, 2\rho, 3\rho, \dots$; arranged in this order. If each fluid subtend an angle 2α at the centre and the tube just holds them all, show that $\tan n\alpha = (2n+1) \tan \alpha$. 5

- (b) Given $u = \frac{-c^2 y}{r^2}$, $v = \frac{c^2 x}{r^2}$, $w = 0$, where r denotes distance from z -axis. Find the surfaces which are orthogonal to streamlines, the liquid being homogeneous. 4

7. (a) Prove that the acceleration of a fluid particle at P is given by

$$\vec{f} = \frac{\partial \vec{u}}{\partial t} + \text{grad} \left(\frac{1}{2} \vec{u}^2 \right) - \vec{u} \times \text{curl } \vec{u}. \quad 4$$

P.T.O.

Handwritten marks:
A small scribble above the word "and".
A circled scribble below the word "and".

(b) Determine the constants l, m, n in order that the velocity $\vec{q} = \{(x+lr)\hat{i} + (y+mr)\hat{j} + (z+nr)\hat{k}\} / \{r(n+r)\}$, where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ may satisfy the equation of continuity of a liquid. 5

8. (a) If a mass of liquid is in equilibrium under a given force \vec{F} whose components per unit mass at the point (x, y, z) parallel to the coordinate axes are (X, Y, Z) , then prove that $\vec{F} \cdot (\nabla \times \vec{F}) = 0$. 4

(b) Find the condition of equilibrium for homogeneous and heterogeneous fluid. 5

$$\int (x dx + y dy + z dz) = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz$$

$$y \rightarrow x(a-z) = ax - zx$$

$$\therefore \frac{\partial y}{\partial z} = a - x \quad \rho x = \frac{\partial P}{\partial x}$$

$$\frac{\partial z}{\partial y} = a \quad y \rightarrow z \quad \left. \frac{\partial P}{\partial y} = \rho y \right\}$$

(9)

DSE 2C

(Portfolio Optimization)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

- (a) What do you mean by risk-free asset?
- (b) By which measure of dispersion we calculate portfolio risk?
- (c) Define Sharpe Ratio.
- (d) What is market index?
- (e) When an Investor agree to invest in high risk investments?
- (f) How expected return (ER) is calculated?
- (g) Write down mathematical formula to calculate the returns from Mutual Funds.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. An investor combine securities M & N and resulting portfolio is risk free. The variance of N is nine times larger than the variance of M . The expected return of

P.T.O.

M & N are 15% & 35% respectively. Find the expected return of the portfolio.

3. Briefly explain capital market line (CML).
4. What is the standard deviation (SD) of a random variable q with the following probability distribution?

<u>Value of q</u>	<u>Probability</u>
0	0.25
1	0.25
2	0.50

5. You have a portfolio with beta of 0.84. What will be the new portfolio beta if you keep 85% of your money in the old portfolio and in a stock with a beta of 1.93.

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) Write down the objective of Investment.
- (b) Suppose security A expected to produce annual return of 18%, 14% and 8% per annum during boom, normal and slump period respectively. Security B expected to earn annual return of 12%, 8%, 8% per annum during boom, normal and slump period. The probability of the economy being in the state of boom, normal and slump in the next period is 30%, 50% and 20% respectively. Explain the diversification with this situation. 2+7

not (a-7)

7. Consider the following information about the return on classic mutual fund, the market return and the treasury bill (T bill) returns.

Year	Return of Classic mutual fund	Market return	T bill return
1994	17.1	10.8	5.4
1995	-14.6	-8.5	6.7
1996	1.7	3.5	6.5
1997	8.0	14.1	4.3
1998	11.5	18.7	4.7
1999	-5.8	-14.5	7.0
2000	-15.6	-26.0	7.9
2001	38.5	36.9	5.8
2002	33.2	23.6	5.0
2003	-7.0	-7.2	5.3
2004	2.9	7.4	6.2
2005	27.4	18.2	10.0
2006	23.0	31.5	11.4
2007	-0.6	-4.9	14.1
2008	21.4	20.4	10.7

Calculate SHARPE RATIO.

9

8. Briefly explain Capital Asset Pricing Model (CAPM). 9

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DSE-1A/1B/1C

Full Marks : 32

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Notations and symbols have their usual meanings.

DSE 1A

(Advanced Algebra)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Prove that $\text{Inn}(S_3) \cong S_3$.

(b) Show that $\langle x^2 + 1 \rangle$ is not a prime ideal of $\mathbb{Z}_2[x]$.

(c) Define Commutator subgroup of a group G .

(d) Prove that for any group G , $|G/Z(G)| \neq 91$.

P.T.O.

(2)

- (e) If H_1, H_2 be normal subgroups of G and G is an internal direct product of H_1 and H_2 , then show that

$$G = H_1H_2 \text{ and } H_1 \cap H_2 = \{e\}.$$

- (f) Prove that every group of order 77 is cyclic.
- (g) Show that the polynomial $2x^5 + 15x^3 + 10x + 5$ is irreducible over \mathbb{Z} .

Group - B

(10 Marks)

Answer any two questions : $5 \times 2 = 10$

2. (a) Let G be a group. Then prove that $\text{Inn}(G)$ is a normal subgroup of $\text{Aut}(G)$.
- (b) Let G be an infinite cyclic group. Then prove that $\text{Aut}(G) \cong \mathbb{Z}_2$. $2+3$
3. (a) Let G be a non-commutative group of order p^3 , where p is a prime. Prove that $|Z(G)| = p$.
- (b) Find all Sylow 3-subgroups of S_4 . $2+3$
4. Show that no group of order 56 is a simple group. 5
5. Let for a group G , $\circ(G) = pq$, where p, q are distinct primes, $p < q$, $p \nmid (q-1)$. Show that G is cyclic. 5

(3)

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) Let for a group G , $\circ(G) = p^n$, p being prime. If H is a subgroup of G such that $\circ(H) = p^{n-1}$, show that H is normal in G . 4
- (b) Let G be a group of order 100. If G has a unique Sylow 2-subgroup, then prove that G is a commutative group. 3
- (c) Let G be a group such that $Aut(G) = \{I_G\}$ where I_G denotes the identity mapping on G . Prove that G is a commutative group and $a^2 = e$ for all $a \in G$. 2
7. (a) Prove that every group of order p^2 , where p is a prime, is commutative. 3
- (b) Determine whether $4 = 1 + 1 + 2$ can be the class equation of a group. Justify your answer. 1
- (c) Prove that every Euclidean domain is a PID. Show that $\mathbb{R}[x]$ is a PID but $\mathbb{Z}[x]$ is not a PID. 3+1
- (d) Let G be a simple group of order 168. Show that G has eight Sylow 7-subgroups. 1

P.T.O.

(4)

8. (a) Find all irreducible polynomials of degree 2 in $\mathbb{Z}_2[x]$. 3
- (b) Examine whether 3 is a prime element in the integral domain $\mathbb{Z}[\sqrt{-3}]$ or not. 4
- (c) If R is an integral domain, then prove that $R[x]$ is also an integral domain. 2
-

(5)

DSE 1B

(Number Theory)

Group - A

(4 Marks)

1. Answer any *four* questions :

1×4=4

- (a) Determine the smallest positive integer having only 10 positive divisors.
- (b) Show that $x^2 + y^2 = 43$ cannot have any integral solution. Explain.
- (c) Find the order of $\bar{5}$ in \mathbb{Z}_{17} .
- (d) For $n = 3000$, find $d(n)$ and $\sigma(n)$.
- (e) Determine $\sum_{j=1}^n \mu(j!)$, where μ is the Möbius function.
- (f) Solve $x^2 \equiv 5 \pmod{29}$.
- (g) Find the remainder when 2^{50} is divided by 7.

P.T.O.

(6)

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Define Euler ϕ -function. Let n be the positive integers of the form $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$, where p_1, p_2, \dots, p_r are distinct primes and each α_i are greater than or equal to 1. Prove that

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_r}\right). \quad 5$$

3. If p is an odd prime and p does not divide a , then $x^2 \equiv a \pmod{p}$ has a solution or no solution depending on whether $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$ or $a^{\frac{p-1}{2}} \equiv -1 \pmod{p}$, respectively. 5

4. Solve the system of linear congruences

$$x \equiv 3 \pmod{11}, x \equiv 5 \pmod{19}, x \equiv 10 \pmod{29}. \quad 5$$

5. Decipher the message

“TZSVIW JQBVMIJ ALMVOOVT”

which was produced using the cipher

$$C = 3p + 7 \pmod{26}. \quad 5$$

(7)

Group - C

(18 Marks)

Answer any *two* questions : $9 \times 2 = 18$

6. (a) If p be a prime number, prove that
 $(p-1)! + 1 \equiv 0 \pmod{p}$. 4

(b) If p be a prime and a is prime to p , prove that

$$a^{p^2-p} \equiv 1 \pmod{p^2}. \quad 2$$

(c) If p is an odd integer, then by using Wilson's theorem show that

$$1^2 \cdot 3^2 \cdot 5^2 \cdot \dots \cdot (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}. \quad 3$$

7. (a) For any positive integer n , show that $n = \sum_{d|n} \phi(d)$.

3

(b) If F and f are two number theoretic functions related by the formula $F(n) = \sum_{d|n} f(d)$, then

show that $f(n) = \sum_{d|n} \mu(d) \cdot F\left(\frac{n}{d}\right)$, where μ is

the Möbius function. Hence show that

$$\phi(n) = \sum_{d|n} \mu(d) \cdot \frac{n}{d}. \quad 3+1=4$$

P.T.O.

(8)

- (c) If (x, y, z) is primitive Pythagorean triplet, prove that both x and y cannot be even integers. 2
8. (a) Show that every prime p has $\phi(p-1)$ primitive roots. 3
- (b) Encrypt the message "NO WAY" by RSA system. 3
- (c) Find the general solution in integers and the least positive integral solutions of the equation $35x - 13y = 10$. 3
-

(9)

DSE 1C

(Bio-Mathematics)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

- (a) What is an endemic?
- (b) Define Allee Effect.
- (c) Find the nature and stability of the fixed point of $\dot{x} = x - by$, $\dot{y} = bx + y$, for different values of b .
- (d) Define Horizontal transmission.
- (e) The system has the characteristic equation
$$\lambda^3 + 4K\lambda^2 + (5 + K)\lambda + 10 = 0.$$
Find the range of K for which the system is stable?
- (f) State the Routh-Hurwitz criteria of order 3.
- (g) Write down a two-species model with diffusion.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Write down the assumptions, draw the schematic diagram and formulate bacterial growth model in a chemostat. 5

P.T.O.

3. What is insect outbreak? Give an example. Write down the insect outbreak model. 5
4. Find the fixed point and investigate their stability for the following logistic map.

$$x_{n+1} = r x_n (1 - x_n), \quad r > 0. \quad 2+3$$

5. Consider the growth model

$$\frac{dN}{dt} = rN \left(\frac{N}{A} - 1 \right) \left(1 - \frac{N}{K} \right),$$

where r , A , K are positive parameters and $A < K$. Determine all the equilibrium points and check their stability. 3+2

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Consider the difference equation

$$x_{n+1} = 0.5x_n \quad \text{with } x_0 = 1024.$$

solve the dynamical system and compute x_{10} .

- (b) Consider the Nicholson-Bailey host-parasitoid model as

$$H_{t+1} = K H_t e^{-aP_t}$$

$$P_{t+1} = c H_t (1 - e^{-aP_t})$$

where H_t and P_t be the host and parasitoid

population size at time t . Here a is the searching efficiency of the parasitoid and c be the number of viable eggs which parasitoid lays on a single host.

Find the fixed points and investigate the stability property of them. 3+6

7. Consider the following system :

$$\frac{dx}{dt} = x(1-x-y)$$

$$\frac{dy}{dt} = \beta(x-\alpha)y.$$

- (a) Show that the system contains no periodic solution within the first quadrant. Here α and β are positive constants.
- (b) Also find the equilibrium points of the system and discuss their stability. 5+4
8. (a) Write down a single-species harvesting model.
- (b) Find the maximum sustainable yield and the optimal harvesting rate.
- (c) Explore transcritical bifurcation if exist. 2+4+3