UG/6th Sem(H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MATH6 - DC-13

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *four* questions : $1\times4=4$

(a) When a feasible solution of an L.P.P called an optimal solution?

(b) State fundamental theorem of L.P.P.

(c) How many basic solutions are possible in a system of m-equations and n-unknowns? $(n \geq m)$

(d) Define a convex set.

(e) State why an assignment problem is not an LPP.

(f) Find the extreme points, if any, of the following set :

$$S = \{(x, y) : x^2 + y^2 \leq 25\}$$

(g) Define saddle point.

P.T.O.

Group - B

Answer any *two* questions : 5×2=10

2. Show that intersection of two convex sets is also a convex set.

3. Solve the following LPP graphically

Maximize $Z = 4x + 2y$

Subject to
$$3x + y \geq 27$$
$$-x - y \leq -21$$
$$x + 2y \geq 30, \quad x, y \geq 0$$

4. Prove that the dual of the dual of a given primal is primal.

5. Solve the following 2 × 4 game problem graphically :

B

		B_1	B_2	B_3	B_4
A	A_1	1	3	0	2
	A_2	3	0	1	–1

Group - C

Answer any *two* questions : 9×2=18

6. Find the solution of the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	10
O_2	13	9	12	8	20
O_3	4	15	7	9	30
O_4	14	7	1	0	40
O_5	3	12	5	19	50
b_j	60	60	20	10	

Examine whether the problem has an alternative optimal solution. 7+2

7. (a) Solve the following L.P.P by simplex method. 5

Maximize $Z = 7x_1 + 5x_2$

Subject to
$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0.$$

(b) Solve the following assignment problem 4

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

P.T.O.

8. (a) Use dominance property to solve the following problem of game 7

B

		B_1	B_2	B_3	B_4
	A_1	4	2	3	2
A	A_2	–2	4	6	4
	A_3	2	1	3	5

(b) Explain the concepts of pure strategies. 2

2023

MATHEMATICS (Honours)

Paper Code : MATH6 - DSE-3(A), 3(B) & 3(C)

Full Marks : 32 Time : Two Hours

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Paper Code : DSE-3(A)

(Point Set Topology)

Group - A

1. Answer any *four* questions : 1×4=4

(a) Let $\mathbb{R}_u$ and $\mathbb{R}_l$ denote respectively the usual topology and the lower limit topology on $\mathbb{R}$. Is the identify function $f : \mathbb{R}_u \to \mathbb{R}_l$ continuous?

(b) Write down a basis for the discrete topology on a non-empty set x.

(c) Find all the limit points of $\{b, c\}$ in the topological space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}\}$.

(d) Find if there exist any set which is neither open nor closed in the topological space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b,c\}\}$.

(e) State continuum hypothesis.

(f) Give example of a path connected topological space.

(g) State the Baire category theorem.

Group - B

Answer any *two* questions : 5×2=10

2. State and prove Schroeder-Bernstein theorem.

3. Let Y be a subspace of a topological space X and A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Show that the closure of A in Y is $\overline{A} \cap Y$.

4. Prove that the union of a collection of connected subspace of a topological space, that have a point in common is connected.

5. Prove that closure of a set is the smallest closed set containing the set.

Group - C

Answer any *two* questions : 9×2=18

6. (a) State Zorn's lemma. Hence prove the Hausdroff Maximal principle. 1+4

(b) Let $\mathcal{B}$ be a basis for a topology τ on X, prove that τ equals to the collection of all unions of elements of $\mathcal{B}$. 4

7. (a) If $\mathcal{B}$ and $\mathcal{C}$ are basis of two topologies on X and Y respectively, then show that the collection $\mathcal{D}=\{B\times C : B\in\mathcal{B}, C\in\mathcal{C}\}$ is a basis for the topology on $X\times Y$. 5

(b) Prove that image of a compact set under a continuous mapping is compact. 4

8. (a) Let $X=\{1,2,3\}$ and $Y=\{a,b,c\}$. Let τ_1 and τ_2 be topologies on X and Y respectively, where $\tau_1=\{\phi, X, \{1\}\}$ and $\tau_2=\{\phi, Y, \{a\}, \{a,b\}\}$. A map f is defined by $1\to a, 2\to a, 3\to b$. Examine whether the mapping f is open, closed, continuous. 5

(b) State and prove the Ascoli-Arzela theorem. 4

P.T.O.

Paper Code : DSE-3(B)

(Theory of Ordinary Differential Equations)

Group - A

1. Answer any *four* questions : 1×4=4

(a) Test for local and global stable point of $\frac{dx}{dt}=x^2-1$.

(b) Prove that two different homogenous systems cannot have the same fundamental matrix.

(c) State the Gronwall's inequality.

(d) Test the stability of the system $\dot{\underset{\sim}{x}}=A\underset{\sim}{x}$ where $A=\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ at its critical point.

(e) If $\underset{\sim}{x}_0 \in \Gamma\underset{\sim}{x}_1$ where $\Gamma\underset{\sim}{x}_2$ is a trajectory of a system passing through $\underset{\sim}{x}_1$ then prove that $\Gamma\underset{\sim}{x}_0=\Gamma\underset{\sim}{x}_1$.

(f) Find the phase paths for the equation

$\ddot{x}+\alpha\sin x=0.$

(g) State maximal interval theorem.

Group - B

Answer any *two* questions : 5×2=10

2. Show that $\frac{f(x,y)\hat{i}+g(x,y)\hat{j}}{\sqrt{[f(x,y)]^2[g(x,y)]^2}}$ gives the vectors of

the vector field for the differential equations $\frac{dx}{dt} = f(x, y)$ and $\frac{dy}{dt} = g(x, y)$. 5

3. Let, $f(t, x)$ be continuous and satisfy a lipschitz condition in $R = \{(t, x) : |t - \tau| \le a, |x - \xi| \le b, (a, b > 0)\}$. Then prove that the IVP $\frac{dx}{dt} = f(t, x)$, $x(\tau) = \xi$ has atmost one solution in $|t - \tau| \le a$. 5

4. Prove that the change of variable $\underset{\sim}{x}(t) = P(t)\underset{\sim}{y}$ transforms the periodic system $\dot{\underset{\sim}{x}} = A(t)\underset{\sim}{x}$ to the system with constant coefficients where $P(t)$ is Floquet normaliser. 5

5. Let A be a $n \times n$ constant matrix. Then prove that a fundamental matrix Φ for $\tilde{x}' = A\tilde{x}$, $(t \in I)$, is given by $\Phi(t) = e^{At}$, $|t| < \infty$. 5

Group - C

Answer any *two* questions : 9×2=18

6. State and prove Cauchy-Peano existance theorem. 9

7. (a) Show that the zero solution of van der Pol's equation $\ddot{x} + e(x^2 - 1)\dot{x} + x = 0$ is uniformly and asymptotically stable when $e < 0$ and unstable when $e > 0$.

   Construct a Lyapunov function for the stable case. 6

P.T.O.

(b) Investigate the stability of the zero solution of the system $\dot{x} = -y - x^3$, $\dot{y} = x - y^3$. 3

8. (a) State and prove Poincare-Benedixon theorem. 5

(b) Diagonalise the coupled linear system

$$\dot{x}_1 = -x_1 - 3x_2$$
$$\dot{x}_2 = 2x_2$$

Hence find the general solution. 4

Paper Code : DSE-3(C)

(Integral Transform)

Group - A

1. Answer any *four* questions : 1×4=4

(a) State and prove first shifting theorem for Laplace transform. 1

(b) Find $L\{t^5 e^{3t}\}$. 1

(c) If $L\{F(t)\} = f(s)$, then prove that

$$L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right).$$ 1

(d) Find $L^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$. 1

(e) Find the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$. 1

(f) Use linearity property of z-transformation to find $z\{\sinh n\theta\}$. 1

(g) Find the Laplace transformation of

$$f(t) = t^n; n > -1$$ 1

P.T.O.

Group - B

Answer any *two* questions : 5×2=10

2. Using Laplace transform, prove that $\int_0^{\alpha} te^{-3t}\sin t\,dt = \frac{3}{50}$. 5

3. Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^{\infty} \frac{x\sin mx}{1+x^2}dx$. 5

4. If $F(x)$ has the Fourier transform $f(s)$, then prove that $F(x)\cos ax$ has the Fourier transform $\frac{1}{2}f(s-a)+\frac{1}{2}f(s+a)$. 5

5. Using residue method or any other method find $Z^{-1}\left\{\frac{9Z^3}{(3z-1)^2(z-2)}\right\}$. 5

Group - C

Answer any *two* questions : 9×2=18

6. (a) Prove that

$$L\left\{\frac{\cos(at)-\cos(bt)}{t}\right\}=\frac{1}{2}\log\left(\frac{p^2+b^2}{p^2+a^2}\right)$$ 4

(b) Solve the integral equation

$$\int_0^{a} F(x)\sin(xt)\,dx = \begin{cases} 1 \ ; & 0 \le t < 1 \\ 2 \ ; & 1 \le t < 2 \\ 0 \ ; & t \ge 2 \end{cases}$$ 5

7. (a) Apply Laplace transform to solve

$\frac{d^2y}{dt^2} + 25y = 10\cos(5t)$ gives that $y = 2$, $\frac{dy}{dt} = 0$

when $t = 0$. 5

(b) Verify convolution theorem in Fourier Transform for the functions

$f(x) = e^{-x^2}$, $g(x) = e^{-x^2}$, $x \in (-\infty, \infty)$. 4

8. (a) Find the Fourier transform of 4

$$f(x) = \begin{cases} 1 \ ; & |x| < a \\ 0 \ ; & |x| > a \end{cases}$$

(b) Using the z-transform, solve the difference equation

$U_{n+2} + 4U_{n+1} + 3U_n = 3^n$ with $U_0 = 0, U_1 = 1$. 5

2023

MATHEMATICS (Honours)

Paper Code : MATH6 - SEC-2

Full Marks : 32 Time : Two Hours

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Group - A

Answer any *four* questions. 1×4=4

1. (a) Let X be a Poisson variate with parameter μ and $P(X=0)=P(X=1)$, prove that $\mu=1$. 1

(b) Find the mean of the random variable X whose density function $f(x)$ is given by 1

$$f(x)\begin{cases} e^{-x} \; ; \; 0<x<\infty \\ 0 \; ; \text{elsewhere} \end{cases}$$

(c) If the lines of regression of y on x and x on y are $3x+2y=26$ and $6x+y=31$ respectively. Find the correlation coefficient between x and y. 1

(d) A die is thrown 108 times in succession. Find the expectation of the number of 'six' appeared. 1

P.T.O.

(e) Find the probability that there may be 53 Sundays in a leap-year. 1

(f) The coefficient of variation is 40 and the mean is 30; find the standard deviation. 1

(g) Define scatter diagram. 1

Group - B

Answer any *two* questions. 5×2=10

2. There are two identical boxes. The first box contains 5 white, 7 red balls and the second box contains 5 white, 5 red balls. One box is chosen at random and a ball is drawn from it. If the ball drawn is found to be white, calculate the probability that it is drawn from the first box. 5

3. Calculate the mean deviation from the mean of the following distribution —

Marks	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

5

4. The scores of two batsmen, A and B, in ten innings during a certain season, are as under —

A :	32	28	47	63	71	39	10	60	96	14
B :	19	31	48	53	67	90	10	62	40	80

Find which of the batsman is more consistent in scoring. 5

5. Let X be a Poisson variate with parameter μ. Show that $P(X \leq n) = \frac{1}{n!}\int_{0}^{\alpha} e^{-x} x^{n} dx$, where n is any positive integer. 5

Group - C

Answer any *two* questions : 9×2=18

6. (a) A coin is tossed $(m + n)$ times $(m > n)$. Show that the probability of getting at least m consecutive heads is $\frac{n+2}{2^{n}+1}$. 5

(b) The I.Q. of students of a class is normally distributed with parameter $m = 100$ and $\sigma = 10$. If the total number of students in the class is 700, then find the number of students who have $I.Q. \geq 115$. Given that $\frac{1}{2\Pi}\int_{-\alpha}^{1.5} e^{\frac{-x^2}{2}} .dx = 0.9332$. 4

7. (a) Find out the skewness and Kurtosis of the series by the method of moments : 5

Measurement	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

P.T.O.

(b) Using the method of least square, fit a curve of the $y = a + bx^2$ to the following data — 4

x	0	1	2	3
y	1	6	20	48

8. (a) For the Binomial (n, p) distribution, prove that

$$\mu_{r+1} = p(p-1)\left[nr\,\mu_{r-1} + \frac{d\mu r}{dp}\right]$$ 5

where μ_r is the rth central moment of the distribution.

(b) If the random variables X and Y are connected by the linear relation $2x + 3y + 4 = 0$. Show that $\rho(x, y) = -1$. 4