LOGARITHM FOR COMMON STUDENTS

Every positive number (e.g. 1, 2, 3... 100, 101, 102.... etc.) can be expressed as a 'power' (ঘাত) of another number. (Recall Index (সূচক) or Power (ঘাত))

For Example

$100 = 10^2$ / $9 = 3^2$ / $128 = 2^7$

$10 = 10^1$ / $81 = 9^2$ or, 3^4 / $256 = 2^8$ or, 4^4

$1 = 10^0$ / $625 = 25^2$ or, 5^4 / $729 = 3^6$ or, 9^3

> Power/Index means the no. of times the number is multiplied.
>
> Eg: $9 = 3^3$ $(3 \times 3 \times 3)$

*This process of expressing any number is called Indices or Index (singular) or power or exponent expression of numbers.

Use of power/Index/Exponent:

Power/index/ exponent is useful to express any large/small number into a consise and convenient form, so that basic mathematical operations (like, multiplication, division, etc.) can be done easily.

Components

When we use power function to any number, it has then have two components.

Example: $1000 = 10^3$

$3 \rightarrow$ Power/Exponent

$10 \rightarrow$ Base.

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Like whole number, decimal numbers can also be expressed as function of power.

Example:

$10^{-1} = 0.1$ or, $\frac{1}{10}$

$10^{-2} = 0.01$ or, $\frac{1}{100}$

$10^{-3} = 0.001$ or, $\frac{1}{1000}$

$2^{-1} = \frac{1}{2}$ or, 0.5

$5^{-2} = \frac{1}{5^2}$ or, 0.04

$8^{-3} = \frac{1}{8^3}$ or, 0.00195

Try more example,

$10^{-5} = ?$

$2^{-3} = ?$

$6^{-6} = ?$

There is a relation between root and power

Reciprocal power is the times root of that number.

Example:

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

$a\sqrt{x} = a \cdot x^{\frac{1}{2}}$

$a\sqrt[3]{x} = a \cdot x^{\frac{1}{3}}$

$\sqrt{x^2} = (x^2)^{\frac{1}{2}}$

$\sqrt[3]{x^4} = (x^4)^{\frac{1}{3}}$

$4\sqrt[3]{x^5} = 4 \cdot (x^5)^{\frac{1}{3}}$

$\sqrt{10} = 10^{\frac{1}{2}}$

$\sqrt[3]{10} = 10^{\frac{1}{3}}$

$\sqrt[8]{9} = 9^{\frac{1}{8}}$

$5\sqrt{9} = 5 \cdot 9^{\frac{1}{2}}$

$6\sqrt[3]{81} = 6 \cdot 81^{\frac{1}{3}}$

$\sqrt{9^2} = (9^2)^{\frac{1}{2}}$

$\sqrt[3]{2^4} = (2^4)^{\frac{1}{3}}$

$4\sqrt[3]{6^5} = 4\,(6^5)^{\frac{1}{3}}$

Try to express Into decimals

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Logarithm or (Log) is the power or exponent which tells us that a given number is what power of another given number.

Example:

$100 = 10^{2}$ ← 2 → Power, 10 → Base

$32 = 2^{5}$ ← 5 → Power, 2 → Base

Number

$\underset{(\text{Base})}{\text{Log}_{10}} \overset{(\text{Number})}{100} = \underset{(\text{Power})}{2}$ / $\underset{(\text{Base})}{\text{Log}_{2}} \overset{(\text{Number})}{32} = \underset{(\text{Power})}{5}$

Question: 100 is which power of 10 **Answer:** 2 $(10^{\boxed{?}} = 100)$

or,

32 is which power of 2 **Answer:** 5 $(2^{\boxed{?}} = 32)$

Log expression: If $a^{x} = N$ where, N = number,
a = base
then, $\text{Log}_{a} N = x$ x = Power (which is the Log output)

Recall power/Index (সূচক)

Base goes at base.
The answer is the number for which Log is desired.
The power is the Log output.

Try Some Examples: $\text{Log}_{3} 81 = 4$ as '$3^{\boxed{4}}$' is exponent function of $\boxed{81}$ and 4 is power

$\text{Log}_{5} 25 = ?$

$\text{Log}_{10} 1000 = ?$

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Rules of Logarithm:

Logarithm is a scientific notation, can be used in mathematical operations more easily. Logarithm follow some rules, which can be used in mathematical operations.

Rule-1 Logarithm of the product of two or more numbers is the sum of their logarithms.

$$\text{Log}(a \times b) = \text{Log}\, a + \text{Log}\, b$$

Example: $\text{Log}(40 \times 8) = \text{Log}\, 40 + \text{Log}\, 8$ [solve it taking base '10']

Rule-2 Logarithm of a fraction is the difference between Log of numerator and log of denominator.

$$\text{Log}\left(\frac{a}{b}\right) = \text{Log}\, a - \text{log}\, b$$

Example:

$$\text{Log}\left(\frac{4}{9}\right) = \text{Log}\, 4 - \text{Log}\, 9$$

$$\text{Log}\left(\frac{3}{7}\right) = \text{Log}\, 3 - \text{Log}\, 7$$

$$\text{Log}\left(\frac{9}{4}\right) = \text{Log}\, 9 - \text{Log}\, 4$$

[Solve taking base 10]

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Rule-3 Logarithm of any number with a certain power is equal to the product of the power and the Log output of the number.

$$\text{Log}(a^b) = b(\text{Log } a)$$

Example:

$\text{Lg}(10^5) = 5 \times \log 10$

$\text{Log}(2^6) = 6 \times \log 2$

$\text{Log}(5^5) = 5 \times \log 5$

[solve taking base 10 use calculator]

Rule-4 Logarithm of any number equal to base is always 1.

$$\text{Log}_a a = 1$$

Example:

$\text{Log}_{10} 10 = 1$

$\text{Log}_5 5 = 1$

$\text{Ine} = 1 \quad \text{Log}_e e = 1$

Rule-5 Logarithm of 1 is always 0 (zero).

$\text{Log}_{10} 1 = 0$ as $10^0 = 1$

$\text{Log}_e 1 = 0$ as $e^0 = 1$

[Try it yourself Use calculator]

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Although, base of Log can be any positive number, common bases used for Logarithm is $\boxed{10}$ and $\boxed{e}$, which is equal to $\boxed{2.71828182\ldots}$

Log with base 10 is Common Logarithm.

Log with base e is Natural Logarithm.

$\boxed{\text{Log}}$. button in calculator

$\boxed{\text{In}}$ button in calculator

$\boxed{\text{Log } 100} = \text{Log}_{10} 100 = 2$ or $100 = 10^2$

$\boxed{\text{In } 100} = \text{Log}_e 100 = 4.60517$ or, $100 = e^{4.60517}$

Calculator

Log 1 = 0.0000000

Log 2 = 0.3010299

Log 3 = 0.4771213

Log 4 = 0.6020599

Log 6 = 0.778151250

Log 8 = 0.90308998

Log 9 = 0.954242509

Log 16 = 1.20411998

×2

×2

×2

Logarithm helps greatly in multiplying and division. For Multiplication it needs simply addition.

For division, it needs simply substraction.

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How to Use Log Table

Log table is a table, which help to find out log output of any given number.

Logarithm of any number consists of

a. Integral part (characteristic)

b. Decimal part (Mantissa)

Example: $\log_{10} 1256 = 3.098989$

3 → Characteristic, 098989 → Mantissa

From Log table, we can find out the mantissa (decimal part). The

The characteristic part is always the number of digits − 1 for the given number for which log is desired.

Example: To find out $\log_{10} \overset{1\,2\,3\,4}{1256}$ the characteristic will be $(4-1) = 3$

How to find out mantissa (decimal part)

4 digits Log table (we can find upto 4 digits)

Find $\log_{10} 1256 = ?$

To study Log table, (which helps getting the decimal parts), first we need to understand its parts from the standpoint of the given number for which log is seek.

Example: For $\log_{10}$ of $\underline{12}\,\underline{5}\,\underline{6}$

Log Table:
- First two digit is the row part.
- Next unit digit is column part
- Next unit digit is mean difference part

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From Log table,

12th row meets 2nd column at $\boxed{0969}$

12th row meets 5th column in mean difference at $\boxed{21}$

$$\boxed{\begin{array}{r} 0969 \\ +\ 21 \\ \hline 0990 \end{array}}$$

Decimal part (Mantissa) will be the sum of two values, which is (0969 + 21 = 0990)

For a number with 3 digit (here 125) characteristic part $\therefore (3-1) = 2$

$\therefore$ Log of $\boxed{1256}$ will be $\boxed{3.09\ .}$ which is actually $\boxed{3.098989}$

Example: $\log_{10} 125.6 = 2.0990$

$\log_{10} 12.56 = 1.0990$

$\log_{10} 1.256 = 0.0990$

Let's try: $\log_{10} 2556 = ?$

$\log_{10} 255.6 = ?$

$\log_{10} 25.56 =$

More Exercise:

$\log_{10} 20 =$

$\log_{10} 55 =$

$\log_{10} 655$

To find out log of any number with digits 2 or 1, add zeros after putting point.

Example: To find out Log 55, try 55.00 and then find row 55 with column 0.

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How to find antilogarithm ?

Till now, we have learnt the Logarithm function,
Let's learn the antilogarithm of a given log output of a given number.

$\boxed{\log_{10} 1000 = 3}$ $\boxed{10^3 = 1000}$ $\boxed{\text{Antilog } 3 = \text{ See antilog table}}$

How to use Antilog table

Like Log table, we can get anti-Log of any four digit number.
We will use the decimal (mantissa) parts.

Example: Antilog of 3.0989 = ?

Digit before point is the characteristic. i.e. $\boxed{3}$ which means $\boxed{\text{4 digit number}}$
Digit after points is mantissa, which is used in antilog table.

$$\underbrace{0}_{\text{Row}}\underbrace{9}_{}$$

09 → Row, 8 → Column, 9 → Mean difference.

09th row meets with 8th column at 1253
09th row meets with 9th column in mean difference at 3

Antilog is the sum of two values.

$$\begin{array}{r} 1253 \\ +\ \ 3 \\ \hline 1256 \end{array}$$

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(iv) *Uses of logarithms*

Logarithms are used to find the product, quotient, powers and roots of numbers.

Example 3 : 21 Find the values of

(a) $439.4 \times .0347 \times 6.8$ (b) $\dfrac{0.6745 \times 8.351}{.00737 \times 4905}$

(e) $35\,(.09)^4\,(.91)^3$ (d) $\dfrac{3.854}{(2.6408)^3}$ (e) $\sqrt[3]{4.8}$

(f) $\dfrac{129}{\sqrt{108 \times 176}}$ (g) $\dfrac{114.72}{\sqrt{548.76 \times 171.20}}$

Solution : (a) $\log(439.4 \times .0347 \times 6.8)$

$$= \log 439.4 + \log .0347 + \log 6.8$$
$$= 2.6429 + \bar{2}.5403 + 0.8325$$
$$= 2.6429 + (-2 + .5403) + 0.8325$$
$$= 2.0157$$

$\therefore\ 439.4 \times .0347 \times 6.8 = \text{antilog } 2.0157 = 103.7$

(b) Let $x = \dfrac{0.6745 \times 8.351}{.00737 \times 4905}$

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(b) Let $x = \dfrac{0.6745 \times 8.351}{.00737 \times 4905}$

$$\begin{aligned}
\log x &= \log (0{\cdot}6745 \times 8{\cdot}351) - \log ({\cdot}00737 \times 4905) \\
&= (\log 0.6745 + \log 8.351) - (\log {\cdot}00737 + \log 4905) \\
&= (\bar{1}.8290 + 0.9218) - (\bar{3}.8675 + 3.6906) \\
&= (-1 + .8290 + .9218) - (-3 + .8675 + 3.6906) \\
&= -1 + 1.7508 - 1.5581 \\
&= -1 + .1927 = \bar{1}.1927
\end{aligned}$$

$\therefore \quad x = \text{antilog } \bar{1}.1927 = 0.1559$

(c) Let $x = 35\,(.09)^4\,(.91)^3$

$$\begin{aligned}
\log x &= \log 35 + \log (.09)^4 + \log (.91)^3 \\
&= \log 35 + 4\,(\log .09) + 3\,(\log .91) \\
&= 1.5441 + 4\,(\bar{2}.9542) + 3\,(\bar{1}.9590) \\
&= 1.5441 + 4\,(-2 + .9542) + 3(-1 + .9590) \\
&= 8.2379 - 11 = .2379 - 3 = \bar{3}.2379
\end{aligned}$$

$x = \text{antilog } \bar{3}.2379 = .001730$

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