Spheroid Geoid and Ellipsoid

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Spheroids, Ellipsoids and Geoids

All map projections start by assuming a particular shape of the Earth. The simplest route would be to assume that the Earth is a perfect sphere (Figure 1). We know this isn't really true, because we can see with our own eyes that the Earth's surface isn't very spherelike at all; its covered with hills, valleys, mountains and so on. However, we know that the Earth's overall shape is fairly spherelike. This leads to a very important concept in geodesy: the idea of breaking the problem of defining the shape of the Earth into two subproblems:

1. First, define the overall shape of the Earth (maybe as a sphere), and
2. Second, superimpose the mountains, valleys and so forth onto the surface of the overall shape.

Spheroids, ellipsoids and geoids all relate to efforts to solve the subproblem of defining the overall shape of the Earth. The second subproblem -- superimposing the mountains, hills, valleys and so forth onto the overall shape of the Earth -- doesn't really enter into the map projection process. Therefore, we can ignore the second subproblem for the time being.

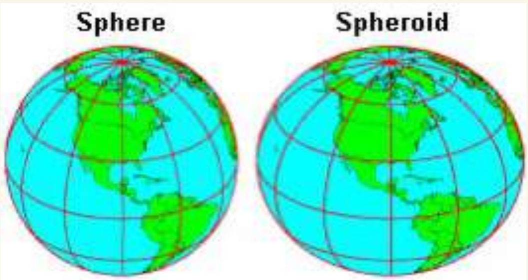


Figure1 Figure. 2

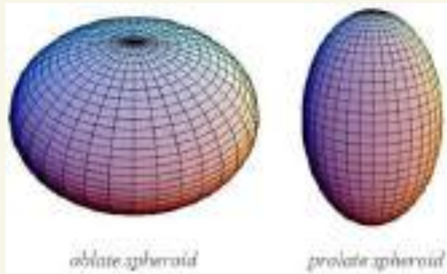
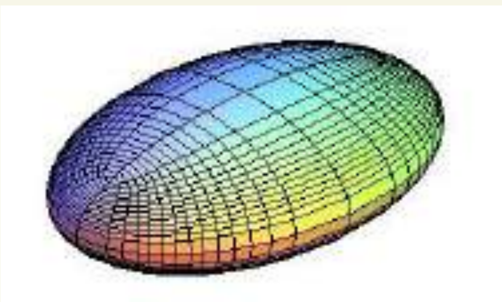


Figure. 2



Ellipsoid Figure. 3

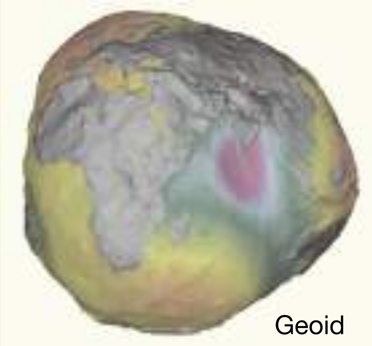
As stated previously, assuming that the Earth is a perfect sphere is the simplest (and the oldest) way of solving this subproblem. In mathematical terms, a true sphere is defined as all points in three-dimensional space located at a specific distance (the radius) from a center point. Thus, in mathematical terms, all you need to know about a sphere is where its center is located and it radius, and you'll know everything there is to know about it. This makes map projection processes based on spheres relatively simple. Unfortunately, the Earth's overall shape really isn't very spherelike, so assuming a spherical Earth isn't very realistic. If you were to produce a map based on this assumption, your map would contain quite a bit of distortion. Therefore, while the spherical Earth assumption was almost universally used in mapmaking a few hundred years ago, modern mapmakers frequently use other assumptions.

Before we entirely dismiss the idea of a sphere, look at Figure 1. In the case of a sphere, the distances from the center to the surface of the sphere are identical along all three axes (e.g., Rx = Ry = Rz). Keep this equality of all three radii in mind as we discuss spheroids and ellipsoids.

In reality, the overall shape of the Earth is more like a spheroid than a true sphere. A spheroid (Figure 2) is a spherelike object where only two of the three radii are of equal length (the third radii can be either longer or shorter than the other two). In reality, the Earth is slightly flattened at the poles, so if you orient a spheroid so that the two equally long radii run through the plane of the equator and the third axis runs through the poles, and shrink the polar axis by about 20 kilometers relative to the other two axes, you get a pretty good idea of the overall shape of the Earth. Over the last 170 years or so, many scientists have developed a number of spheroids that have been widely used in mapmaking.

One note: "Spheroid" is actually not the correct mathematical term for the shapes we've been discussing. Technically, they should be called ellipsoids of revolution. However, the term "spheroid" has become well established in geodesy, and we'll use this term throughout the remainder of this presentation. Just don't get confused if a mathematician tells you that your spheroids are really ellipsoids of revolution!

Spheroids are very popular models of the overall shape of the Earth. However, they are not the only game in town: ellipsoids are another possible model. As you have probably already guessed, an ellipsoid is a spherelike object where all



three axes are of differing length (Figure 3). However, ellipsoids don't do an appreciably better job of representing the overall shape of the Earth than do spheroids, and the mathematics involved in using ellipsoids is considerably more difficult than the math involved in using spheroids. Thus, ellipsoids are rarely used in modern geodesy. To make matters even more complicated, some geodetic scientists use the terms spheroid and ellipsoid interchangeably. While this is not technically correct, it isn't as much of a problem as you might expect, because true ellipsoids are rarely used. However, it is a potentially confusing and sloppy habit, so we suggest you strive to be accurate, and call a spheroid a spheroid, and an ellipsoid an ellipsoid.

No matter how much we juggle with the lengths and orientations of the radii of spheroids and ellipsoids, no spheroid or ellipsoid is going to be a completely accurate representation of the overall shape of the Earth. In reality, the overall shape of the Earth is very complex, with lots of undulations and irregularities. This brings us to a second important concept in geodesy; namely, the idea of geoids.

In priciple, the idea of a geoid is quite simple. A geoid is nothing more than the true overall shape of the Earth. Unfortunately, in practice, geoids are a lot more complex than what this simple definition suggests. Mostly, this complication arises from the fact that in order to come up with a precise definition of a geoid, we need to be a lot more precise about what we mean by the "overall shape" of the Earth.

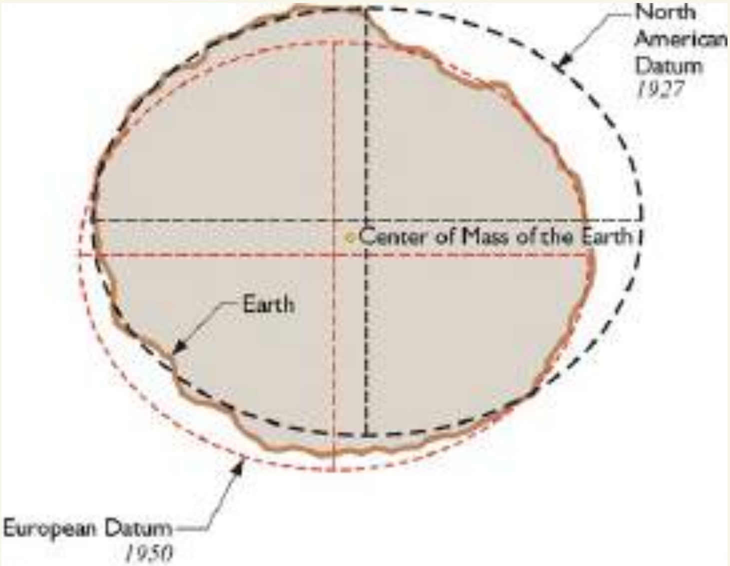
The technical definition of a geoid states that at all points, the geoid's surface is located at a distance from the Earth's center of mass where an defined standard object weighs exactly a specified amount. This sounds ridiculous, but there is a method to the madness.

The weight of any object on Earth is determined by the force of gravity acting between the object and the Earth itself. The stronger the force of gravity, the greater the weight of the object. The force of gravity between any two objects (like the standard object and the Earth) is calculated by adding the masses of the objects and dividing by the squared distance between the centers of mass of the two objects. For you mathematical types, the formula is (M1 + M2) / D212, where M1 and M2 are the masses of objects 1 and 2, respectively, and D212 is the squared distance between the centers of mass of objects 1 and 2. Thus, if you placed the standard object a billion miles from the center of mass of the Earth, the D212 part of the equation would become huge, so the overall force of gravity between the Earth and the object would be very small, and the standard object would weigh practically nothing. Conversely, if you placed the standard object very close to the Earth's center of mass, the D212 part of the equation would become very small, so the overall force of gravity between the object and the Earth would be very high, and the standard object would weigh a great deal. The point here is that weights are relative to distances.

The geoid is defined in such a way that the standard object weighs exactly the same amount at all points on the geoid's surface (this characteristic makes a geoid what is termed an equipotential surface). If the Earth's mass was uniformly distributed throughout the planet, the geoid would be a perfect sphere. However, since the Earth's mass is not evenly distributed, the surface of the geoid undulates. This uneven distribution of mass gives the geoid its complex shape.

No one knows the exact shape of the geoid. It is an extremely complex shape; using sophisticated satellite and statistical techniques, the U.S. Defense Mapping Agency tried to develop an exact mathematical model of the geoid, and gave up after developing a model with 32,755 coefficients and terms ranging up to the 180th power. However, even though it's exact shape is unknown, the geoid remains a very useful idea: It is the "true" overall shape of the Earth, and thus it represents the perfect shape that spheroids, ellipsoids and other models should aspire to reach.

Thus, we are left with three shapes we must deal with (Figure 4). First, we have the spheroid, which is an imperfect model of the overall shape of the Earth, but we know its dimensions exactly and hence we can use it in the projection process. Second, we have the geoid, which is the shape we would like to use in the projection process, but we can't because we don't know its true dimensions (and it is incredibly complex). Finally, we have the shape of the true surface of the Earth, which is infinitely complex and hence impossible to model exactly, but which is, after all, the surface our maps are supposed to represent.



Relationship between a Ellipsoid, Geoid and actual or surface as Datum