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Common Fixed Point Theorem in Probabilistic Metric Space Using Lukasiecz t-norm and Product t-norm

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Abstract: In this paper, we investigate some common fixed point theorems in probabilistic metric spaces. Also, we introduce the concept of point-wise R-weakly commuting pair of self mappings and compatible pair of mappings.

Keywords: Probabilistic metric space, t-norm, Reciprocal continuity, Compatible mappings, R-weakly commuting mappings.

1 Introduction and Preliminaries

In fixed point theory, contraction mapping theorems have always been an active area of research since 1922 with the celebrated Banach contraction fixed point theorem $\{cf.[1]\}$. The notion of probabilistic metric space (briefly, PM-space) was first introduced by Menger [2] in 1943. After that, Schweizer and Sklar [3] developed some fixed point theory in the later part of 1983. Several contraction mapping theorems for commuting mappings have been proved in PM-spaces $\{cf.$ [4],[5],[6],[7] and [8]$\}$. For the convenience of our work, we first introduce the following definitions and notations.

Definition 1.*[3] A Probabilistic Metric Space (PM-space) is an ordered pair (X,F), where X is a non-empty set of elements and F is probabilistic distance on X such that $F_{x,y}$ satisfies the following conditions:*
(PM1) $F_{x,y}(t) = 1$ for all $t > 0$ iff $x = y$;
(PM2) $F_{x,y}(0) = 0$;
(PM3) $F_{x,y}(t) = F_{y,x}(t)$ and
(PM4) If $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$ then $F_{x,z}(t+s) = 1$ for all $x,y,z \in X$ and $t,s \geq 0$.

PM-spaces can be obtained from any metric space by simply introducing the distribution function in that space. Thus, PM-space is a wide area of research compared to a metric space. Schweizer and Sklar [3] defined triangular norm (briefly t-norm) and gave the examples of four basic t-norm, namely, $\Delta_D, \Delta_L, \Delta_P,$ and Δ_M.

As regards the pointwise ordering, we have the relation $\Delta_D < \Delta_L < \Delta_P < \Delta_M$. We are familiar with the notions of commutative mappings, weakly commuting mappings, compatible mappings and weakly compatible mappings in the light of general metric spaces. Similar types of mappings including pointwise R-weakly commuting mappings and reciprocal continuity of mappings were defined by several Mathematicians in PM-space with the introduction of distribution functions.

*Example 1.*Let (X,d) be a metric space where $X = [0,a]$ and let

$$F_{x,y}(t) = \begin{cases} 0, \text{ if } t = 0 \\ 1, \text{ if } t = a \\ \frac{t}{t+d(x,y)}, \text{if } 0 < t < a, \end{cases}$$

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for all $x,y \in X$. Then (X,F) is a PM-space.

In 2011, Modi and Pal [9] investigated some interesting theorems in PM-Space. In 2008, Kumar and Pant [10] investigated a very important lemma, some common fixed point theorems and some corollaries satisfying contractive conditions with an implicit relation. In the results of Kumar and Pant [10], Φ denotes the class F_4, the family of all real valued continuous functions $F:(\mathbb{R}^+)^4 \to \mathbb{R}$.

Now, the following questions arise: What will happen if we introduce lukasiecz $t-$norm in place of max function in Theorem 3.1 of Modi and Pal [9] and product $t-$norm in place of $\phi-$function in Lemma 4.1 and Theorem 4.1 of Kumar and Pant [10]? The present paper aims to answer the above-mentioned questions. Here, we obtain the following common fixed point theorems by point-wise R-weakly commuting pair of self mappings and weakly compatible condition satisfying contractive conditions by improving the results of Modi and Pal [9] as well as Kumar and Pant [10]. Some concepts for modification have been taken from [11] and [12].

2 Main Results

In this section, we prove some theorems and lemmas to modify the results of Modi and Pal [9] as well as Kumar and Pant [10]. Also, Lemma 2.1 of [9] and the Lemma 2 of [13] are the effective tools throughout our work.

Theorem 1.*Let A,B,P and Q be self maps on a probabilistic metric space satisfying*

(a) $P(X) \subset B(X), Q(X) \subset A(X)$;

$(b)$$F_{Px,Qy}(kt) \geq \max\{F_{Ax,By}, \Delta_L(F_{Px,Ax}(t), F_{Qx,Bx}(kt))\}$ for all $x,y \in X$, $t>0$ and $k \in (0,1)$ where Δ_L is Lukasiecz t-norm;

(c) If one of $P(X), B(X), Q(X)$ and $A(X)$ is complete subset of X, then

(i) P and A have a coincident point. (ii) Q and B have a coincident point and if the pair (P,A) and (Q,B) are weakly compatible, then A,B,P and Q have a unique common fixed point.

Proof.: Since $P(X) \subset B(X)$ and $Q(X) \subset A(X)$, there exist sequnces $\{x_n\}, \{y_n\} \in X$ such that $y_{2n+1} = P_{x_{2n}} = B_{x_{2n+1}}, y_{2n+2} = Q_{x_{2n+1}} = A_{x_{2n+2}}$ for all $n = 0,1,2,\cdots$. By (b) we have

$$\begin{aligned} F_{P_{x_{2n}},Q_{x_{2n+1}}}(kt) &\geq \max\left\{F_{A_{x_{2n}},B_{x_{2n+1}}}(t), \Delta_L(F_{P_{x_{2n}},A_{x_{2n}}}(t), F_{Q_{x_{2n}},B_{x_{2n}}}(kt))\right\} \\ \text{i.e., } F_{y_{2n+1},y_{2n+2}}(kt) &\geq \max\left\{F_{y_{2n},y_{2n+1}}(t), \max(F_{y_{2n+1},y_{2n}}(t) + F_{y_{2n+1},y_{2n}}(kt) - 1), 0\right\} \\ &\Rightarrow F_{y_{2n+1},y_{2n+2}}(kt) \geq F_{y_{2n},y_{2n+1}}(t). \end{aligned}$$

Similarly, we can obtain that

$$F_{y_{2n+2},y_{2n+3}}(kt) \geq F_{y_{2n+1},y_{2n+2}}(t).$$

In general, for any n and t, we get that

$$F_{y_n,y_{n+1}}(kt) \geq F_{y_{n-1}},y_n(t).$$

Hence, by Lemma 2.1 of [9], we get a Cauchy sequence $\{y_n\} \in X$.

By completeness, we get $y_n \to z \in X$. Thus, the subsequences $(y_{2n}), (y_{2n+1})$ and (y_{2n+2}) also converges to z. Therefore, $B_{x_{2n+1}}, P_{x_{2n}}, Q_{x_{2n+1}}$ and $A_{x_{2n}}$ also converges to z. Now suppose $A(X)$ is complete. Here, $A(X)$ contains the subsequence (y_{2n+2}) which converges to a point, say z in $A(X)$. Let $w \in A^{-1}(z)$, then $Aw = z$. Then, from (b), we get that

$$\begin{aligned} F_{Pw}, Qx_{2n+1}(kt) &\geq \max\left\{F_{Aw,Bx_{2n+1}}(t), \max\{F_{Pw,Aw}(t) + F_{Qw,Bw}(kt) - 1, 0\right\} \\ i.e., F_{Pw}, y_{2n+1}(kt) &\geq \max\left\{F_{Aw,y_{2n+1}}(t), \max(F_{Pw,Aw}(t) + F_{Qw,Bw}(kt) - 1, 0)\right\} \\ i.e., F_{Pw,z}(kt) &\geq \max\left\{F_{z,z}(t), \max(F_{Pw,Aw}(t) + F_{Qw,Bw}(kt) - 1, 0)\right\} \text{when } n \to \infty \\ &= F_{z,z}(t) = 1. \end{aligned}$$

Therefore, $Pw = z$. Since $Aw = z$, w is a coincident point of P and A. Again, from $P(X) \subset B(X)$ and $Pw = z$, we get $z \in B(X)$. Also, let $v \in B^{-1}(z)$, then $Bv = z$. By (b),

$$\begin{aligned} F_{Px_{2n},Qv}(kt) &\geq \max\left\{F_{Ax_{2n},Bv}(t), \max(F_{Px_{2n},Ax_{2n}}(t) + F_{Qx_{2n},Bx_{2n}}(kt) - 1, 0)\right\} \\ \text{i.e., } F_{y_{2n+1},Qv}(kt) &\geq \max\left\{F_{y_{2n},Bv}(t), \max(F_{y_{2n+1},y_{2n}}(t) + F_{y_{2n+1},y_{2n}}(kt) - 1, 0)\right\} \\ \text{i.e., } F_{z,Qv}(kt) &\geq \max\left\{F_{z,z}(t), \max(F_{z,z}(t) + F_{z,z}(kt) - 1, 0)\right\}, \text{when } n \to \infty \\ \text{i.e., } F_{z,Qv}(kt) &\geq F_{z,z}(t) = 1. \end{aligned}$$

So, $F_{z,Qv}(kt)=1 \Rightarrow Qv=z$. Since $Bv=z$, v is a coincident point of Q and T.

Therefore, P and A commute at their coincident point being (P,A) is weakly compatible and hence $PA\omega=AP\omega$ or $Pz=Az$. Also, the pair (Q,B) is weakly compatible. Therefore, $QBv=BQv$ or $Qz=Bz$. By (b), we have

$$F_{Pz,Qx_{2n+1}}(kt)\geq \max\left\{F_{Az,Bx_{2n+1}}(t),\max(F_{Pz,Az}(t)+F_{Qz,Bz}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{Pz,y_{2n+2}}(kt)\geq \max\left\{F_{Az,y_{2n+1}}(t),\max(F_{Pz,Az}(t)+F_{Qz,Bz}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{Pz,z}(kt)\geq \max\left\{F_{z,z}(t),\max(F_{Pz,Az}(t)+F_{Qz,Bz}(kt)-1;0)\right\}, \text{when } n\to\infty$$
$$\text{i.e., } F_{Pz,z}(kt)\geq \max\{F_{z,z}(t),1\} \text{ i.e., } F_{Pz,z}(kt)\geq 1.$$

Then, $Pz=z$. Similarly, $Qz=z$. Again by (b)

$$F_{Px_{2n},Qz}(kt)\geq \max\left\{F_{Ax_{2n},Bz}(t),\max(F_{Px_{2n},Ax_{2n}}(t)+F_{Qx_{2n},Bx_{2n}}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{y_{2n+1},Qz}(kt)\geq \max\left\{F_{y_{2n},Bz}(t),\max(F_{y_{2n+1},y_{2n}}(t)+F_{y_{2n+1},y_{2n}}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{z,Qz}(kt)\geq \max\left\{F_{z,Bz}(t),\max(F_{z,z}(t)+F_{z,z}(kt)-1,0)\right\}, \text{when } n\to\infty$$
$$\text{i.e., } F_{z,Qz}(kt)\geq \max\{F_{z,Bz}(t),1\}$$
$$\text{i.e., } F_{z,Qz}(kt)=1.$$

Then, $Qz=z$. Therefore, z is a common fixed point of the mappings A,B,P and Q.

Uniqueness:

For uniqueness, we consider the point ω as another common fixed point. Then, by (b), we get

$$F_{P\omega,Qz}(kt)\geq \max\left\{F_{A\omega,Bz}(t),\max(F_{P\omega,A\omega}(t)+F_{Q\omega,B\omega}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{P\omega,Qz}(kt)\geq \max\left\{F_{\omega,z}(t),\max(F_{\omega,\omega}(t)+F_{\omega,\omega}(kt)-1,0)\right\}$$
$$\text{i.e., } F_{\omega,z(kt)}\geq \max\left\{F_{\omega,z}(t),1,0\right\}\Longrightarrow F_{\omega,z}(kt)=1\Longrightarrow \omega=z.$$

Hence, z is only common fixed point of the theorem.

This completes the proof of the theorem.

Lemma 1.*Let (X,F,Δ_M) be a complete Menger space. Furthermore, let (A,S) and (B,T) be point-wise $R-$weakly commutating pair of self mapping of X satisfying*

(I) $A(X)\subseteq T(X), B(X)\subseteq S(X)$;

(II) $\Delta_P(F_{Au,Bv}(ht),F_{Bv,Tv}(ht))\geq \Delta_P(F_{Su,Tv}(t),F_{Au,Su}(t))$ for all $u,v\in X, t>0, h\in(0,1)$ and Δ_P is the product t-norm on Δ. Then, the continuity of one of the mapping in the compatible pair (A,S) or (B,T) on (X,F) implies their reciprocal continuity.

Proof.: First assume that S is continuous and the pair A and S are compatible. Let $\{u_n\}$ be a sequence such that $Au_n\to z$ and $Su_n\to z$ where $z\in X$ as $n\to\infty$. Since S is continuous, we have $SAu_n\to Sz$ and $SSu_n\to Sz$ as $n\to\infty$ and since (A,S) is compatible, we get $F_{ASu_n,SAu_n}(t)\to 1$, implies $F_{ASu_n,Sz}(t)\to 1$ or $ASu_n\to Sz$ when $n\to\infty$.

Using (I), we get some $v_n\in X$ for which $ASu_n=Tv_n$. Hence, $SSu_n\to Sz$, $SAu_n\to Sz$, $ASu_n\to Sz$ and $Tv_n\to Sz$ whenever $ASu_n=Tv_n$. Now, we assert that $Bv_n\to Sz$ as $n\to\infty$. Then, by (II), we have

$$\Delta_P(F_{ASu_n,Bv_n}(ht),F_{Bv_n,Tv_n}(ht))\geq \Delta_P(F_{SSu_n,Tv_n}(t),F_{ASu_n,SSu_n}(t))$$
$$\text{i.e., } \Delta_P(F_{S,Bv_n}(ht),F_{Bv_n,Sz}(ht))\geq \Delta_P(F_{Sz,Sz}(t),F_{Sz,Sz}(t))$$
$$\text{i.e., } F_{Bv_n,Sz}(ht)\cdot F_{Bv_n,Sz}(ht))\geq F_{Sz,Sz}(t)\cdot F_{Sz,Sz}(t).$$

Where $\Delta_P(x,y)=x\cdot y$ and $F_{x,y}(t)=F_{y,x}(t)$. Therefore, we have $[F_{Bv_n,Sz}(ht)]^2\geq 1\cdot 1=1$ i.e., $F_{Bv_n,Sz}(ht)\geq 1$, for all $t>0$. Thus, we have $F_{Bv_n,Sz}(ht)=1$ *i.e.*, $Bv_n\to Sz$.

Again by (II), we have $\Delta_P(F_{Az,Bv_n}(ht),F_{Bv_n,Tv_n}(ht))\geq \Delta_P(F_{Sz,Tv_n}(t),F_{Az,Sz}(t))$. By taking limit as $n\to\infty$, we get that

$$F_{Az,Sz}(ht)\cdot F_{Sz,Sz}(ht)\geq F_{Sz,Sz}(t)\cdot F_{Az,Sz}(t)$$
$$\text{i.e., } F_{Az,Sz}(ht)\geq F_{Az,Sz}(t),$$

which implies that $Az=Sz$, by Lemma 2 of [13]. Thus, $S_{Au_n}\to Sz$ and $ASu_n\to Sz=Az$ when $n\to\infty$, implies the reciprocal continuity of A and S on X. Applying similar procedure, we can prove that B and T are reciprocally continuous when T is continuous.

This proves the lemma.

Theorem 2. *Let (X,F,Δ_M) be a complete Menger space. Moreover, let (A,S) and (B,T) be point-wise R-weakly commuting pair of self mapping of X satisfying*

(I) $A(X) \subseteq T(X), B(X) \subseteq S(X)$.

(II) $\Delta_P(F_{Au,Bv}(ht), F_{Bv,Tv}(ht)) \geq \Delta_P(F_{Su,Tv}(t), F_{Au,Su}(t))$ for all $u,v \in X, t > 0, h \in (0,1)$ and P is the product $t-norm$ on Δ.

If one of the mappings in compatible pair (A,S) or (B,T) is continuous, then A,B,S and T have a unique common fixed point.

Proof.: Let $u_0 \in X$. By condition (I), we define the sequence $\{u_n\}$ and $\{v_n\}$ in X such that for all $n = 0,1,2,\cdots$, we have

$$\text{(a) } v_{2n+1} = Au_{2n} = Tu_{2n+1}, v_{2n+2} = Bu_{2n+1} = Su_{2n+2}.$$

Again by (II), we get that

$$\Delta_P(F_{Au_{2n},Bu_{2n+1}}(ht), F_{Bu_{2n+1},Tu_{2n+1}}(ht)) \geq \Delta_P(F_{Su_{2n},Tu_{2n+1}}(t), F_{Au_{2n},Su_{2n}}(t))$$

$$\Rightarrow \Delta_P(F_{v_{2n+1},v_{2n+2}}(ht), F_{v_{2n+2},v_{2n+1}}(ht)) \geq \Delta_P(F_{v_{2n};v_{2n+1}}(t), F_{v_{2n+1},v_{2n}}(t))$$

$$\Rightarrow F_{v_{2n+1},v_{2n+2}}(ht) \cdot F_{v_{2n+1},v_{2n+2}}(ht)) \geq F_{v_{2n},v_{2n+1}}(t) \cdot F_{v_{2n},v_{2n+1}}(t)$$

$$\Rightarrow [F_{v_{2n+1},v_{2n+2}}(ht)]^2 \geq [F_{v_{2n},v_{2n+1}}(t)]^2,$$

which implies that (b) $F_{v_{2n+1},v_{2n+2}}(ht) \geq F_{v_{2n},v_{2n+1}}(t)$, since $F_{x,y}$ is non-decreasing and $F_{x,y} \geq 0$. Again by condition (II), we have

$$\Delta_P(F_{Au_{2n+1},Bu_{2n+2}}(ht), F_{Bu_{2n+2},Tu_{2n+2}}(ht)) \geq \Delta_P(F_{Su_{2n+1},Tu_{2n+2}}(t), F_{Au_{2n+1},Su_{2n+1}}(t))$$

$$\Rightarrow F_{v_{2n+2},v_{2n+3}}(ht) \cdot F_{v_{2n+3},v_{2n+2}}(ht) \geq F_{v_{2n+1},v_{2n+2}}(t) \cdot F_{v_{2n+2},v_{2n+1}}(t)$$

$$i.e., F_{v_{2n+3},v_{2n+2}}(ht) \geq F_{v_{2n+1},v_{2n+2}}(t).$$

Also, $F_{v_n,v_{n+1}}(ht) \geq F_{v_{n-1},v_n}(t)$ for all n and t. Hence, by Lemma 2.1 of [9], $\{v_n\}$ is a Cauchy sequence in X. Since X is complete, $\{v_n\}$ converges to z. It's subsequences $\{Au_{2n}\}$, $\{Bu_{2n+1}\}$, $\{Su_{2n}\}$ and $\{Tu_{2n+1}\}$ also converge to z.

Now, we consider that S is continuous and (A,S) is compatible pair. Then, using Lemma 1, A and S are reciprocally continuous, so $ASu_{2n} \to Az$, $SAu_{2n} \to Sz$. Also, compatibility of A and S gives $F_{ASu_{2n},SAu_{2n}}(t) \to 1$, therefore $F_{Az,Sz}(t) \to 1$ as $n \to 1$. Hence, $Az = Sz$. Since $A(X) \subseteq T(X)$, there exists a point p in X for which $Az = Tp$.

By condition (II), we have

$$\Delta_P(F_{Az,Bp}(ht), F_{Bp,Tp}(ht)) \geq \Delta_P(F_{Sz,Tp}(t), F_{Az,Sz}(t)),$$

$$\Rightarrow F_{Az,Bp}(ht) \cdot F_{Bp,Az}(ht) \geq F_{Sz,Az}(t) \cdot F_{Az,Sz}(t)$$

$$\Rightarrow [F_{Az,Bp}(ht)]^2 \geq 1, \text{ since } Az = Sz$$

$$\Rightarrow FAz,Bp(ht) \geq 1, \text{ for all } t > 0 \text{ and } h \in (0,1),$$

which gives $F_{Az,Bp}(ht) = 1$, so $Az = Bp$. Thus, $Az = Sz = Bp = Tp$. Since A and S are point-wise R-weakly commuting mapping, there exists an $R > 0$ such that

$$F_{ASz,SAz}(t) \geq F_{Az,Sz}(t/R) = 1,$$

so we have $ASz = SAz$ and $AAz = ASz = SAz = SSz$. Similarly, since B and T are point-wise R-weakly commuting mapping, we have $BBp = BTp = TBp = TTp$.

Again by condition (II), we have

$$\Delta_P(F_{AAz,Bp}(ht), F_{Bp,Tp}(ht)) \geq \Delta_P(F_{SAz,Tp}(t), F_{AAz,SAz}(t))$$

$$\text{i.e., } F_{AAz,Az}(ht) \cdot F_{Az;Az}(ht) \geq F_{AAz,Az}(t) \cdot F_{AAz,AAz}(t)$$

$$\Rightarrow F_{AAz,Az}(ht) \cdot 1 \geq F_{AAz,Az}(t) \cdot 1$$

$$\Rightarrow F_{AAz,Az}(ht) \geq F_{AAz,Az}(t).$$

This gives $AAz = Az$, implying that $Az(= AAz = SAz)$ and $Bp(= Az)$ are the common fixed points of A,S and B,T respectively and for all mappings A,B,S and T.

Uniqueness:

Finally, suppose that $Ap(\neq Az)$ is another fixed point. By condition (II),

$$P\left(F_{AAz,BAp}(ht), F_{BAp,TAp}(ht)\right) \geq \Delta_P(F_{SAz,TAp}(t), F_{AAz,SAz}(t)),$$

$$\text{i.e., } F_{Az,Ap}(ht) \cdot F_{Ap,Ap}(ht) \geq F_{Az,Ap}(t) \cdot F_{Az,Az}(t),$$

$$\text{i.e., } F_{Az,Ap}(ht) \cdot 1 \geq F_{Az,Ap}(t) \cdot 1,$$

$$\text{i.e., } F_{Az,Ap}(ht) \geq F_{Az,Ap}(t).$$

Which gives that $Az = Ap$ and Az is a unique common fixed point.

This completes the proof of the theorem.

3 Conclusion

In the line of the works as carried out in the paper one may think of the deduction of fixed point theorems using fuzzy metric, quasi metric, partial metric and other different types of metrics under the flavour of probabilistic metric space. This may be an active area of research to the future workers in this branch.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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